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Abstract of the invited 37th Annual Gödel Lecture

▶ JOHN P. BURGESS, *Logical and mathematical conventionalism.*
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The 1930 Königsberg conference began with a symposium on logicism, intuitionism, and formalism, but also included other events at one of which Gödel announced his first incompleteness theorem. Its importance was not instantly recognized on that occasion, but it was soon to create difficulties for all three leading schools. Yet even after those three schools faded, something of the spirit of logicism lived on among philosophers in conventionalism, of which Carnap, who had represented logicism at the symposium, became the most influential advocate. Among philosophers, if not logicians, conventionalism was arguably something like the received view right through the 1950s, until criticism by Quine and others eventually led to its being universally abandoned. Recently, however, there has been a revival of conventionalism, both about logic and about mathematics, and some highlights of recent developments will be addressed in this lecture.

Abstract of invited tutorials

▶ PHILIPP HIERONYMI, *Logic, geometry, and computation: A survey of expansions of additive structures.*
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We survey recent results on expansions of Presburger arithmetic and of the ordered additive group of the reals. Such structures have long been studied by logicians and theoretical computer scientists for a variety of reasons. In this survey, we provide an overview of recent developments in the area and explain their relevance to model theory, theoretical computer science, and tame geometry.

This series of talks aims partially update and connect the following earlier surveys: "A Survey of Arithmetical Definability" by Alexis Bes, "Open questions around Büchi and Presburger arithmetics" by Christian Michaux and Roger Villemaire and "Tameness in expansions of the real field" by Chris Miller.

▶ RAHUL SANTHANAM, *Provability of lower complexity bounds.*
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To be announced.

Abstracts of invited plenary lectures

▸ JUAN PABLO AGUILERA, *The kappa-Turing degrees.*
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We study a generalization of the recursively enumerable sets defined in terms of Σ-1-definable functionals at admissible levels of Gödel's constructively hierarchy. These sets and the corresponding notion of reducibility give rise to a new kind of degree structure for subsets of non- projectible admissible ordinals. This is joint work with N. Greenberg and E. Hammatt.

▸ CAROLIN ANTOS, *Conceptual change in set theory - between continuity and revolution..*
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The general question of whether revolutions are possible in mathematics remains contested. Prominently, Crowe (1975, 165) claims that revolutions cannot occur in mathematics as revolutions require that "a previously accepted entity within mathematics proper be rejected." In Kuhn's account, such discontinuities are a significant feature of scientific revolutions. If one wants to argue against Crowe that change in mathematics is comparable to change in the (natural) sciences, one can either show that discontinuities do indeed exists in mathematical change (see Kitcher (1984)), or that change in the natural sciences is not as discontinuous as Kuhn claims it to be (see Post (1971)).

In this talk, I will use these possible answers to study a change that took place in the 1960's within the field of set theory. Here, the introduction and development of model-theoretic methods, such as ultrapower constructions, inner models and forcing, sparked an explosion of results, clarified long-standing questions and opened up new areas of research. Most importantly, these methods provided a unified methodology for set theory's two main epistemic functions: serving as a foundational theory on the one hand, and studying mathematical infinity on the other.

This model-based turn in set theory exhibits several markers of revolutionary theory change: It addressed not only a lack of progress in some areas of set theory, but it resolved a fundamental flaw of set theory at that time, namely the lack of a unified methodological basis. This new basis led to an abandonment of earlier methods and explanatory patterns, as well as changing the general understanding of fundamental concepts and research questions. Are these markers sufficient to call the change in set theory a revolution?

[1] Michael J. Crowe, *Ten "laws" concerning patterns of change in the history of mathematics*, ***Historia Mathematica***, vol. 2 (1975), no. 2, pp. 161–166.

[2] Philip Kitcher, ***The Nature of Mathematical Knowledge***, Oxford University Press, 1984.

[3] Heinz R. Post, *Correspondence, Invariance and Heuristics: In Praise of Conservative Induction*, ***Studies in History and Philosophy of Science Part A***, vol. 2 (1971), no. 3.

▸ LIRON COHEN, *The effects of effects on constructivism.*
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Constructivism begins with the promise that truth has computational meaning. Yet our standard picture of constructive validity grew out of a pure model of computation, inherited from the lambda calculus. Is this purity a foundational necessity, or merely a historical convenience? Today, computation is effectful: programs store state, branch nondeterministically, raise exceptions, interact with environments, sample randomly, and manipulate continuations. If constructive foundations are meant to explain computation, rather than idealize it away, then effects must move from the margins into the logical core. In this talk, I will argue that effects are not obstacles to constructivism. They are the structure that a modern constructivism should be built from. Once effects are admitted, they reshape the validity of logical principles: sometimes breaking them, sometimes restoring them, and often revealing distinctions that pure foundations hide. I will illustrate this through examples involving choice, continuity, and Markov's principle. What emerges is not a weaker constructivism, but a more honest one: a constructivism whose logical principles are grounded in the actual structure of computation.

▸ SU GAO, *The isomorphism relation of non-Archimedean Polish groups*.
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Kechris, Nies and Tent proposed a program to study the isomorphism relation of Borel classes of non-Archimedean Polish groups and to determine their exact complexity in the Borel reducibility hierarchy. In this talk I will give a survey of the known results in this program, including some recent work on extremely amenable groups and procoutable groups. This involves joint work with Wei Dai, Mahmood Etedadialiabadi, Feng Li, Ruiwen Li, Andre Nies, Gianluca Paolini, and Víctor Hugo Yañez.

▸ ÅSA HIRVONEN, *Ultraproduct approximations in metric model theory*.
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Continuous logic offers an approach to metric structures that in many aspects resembles classical first order model theory. As for ultraproducts, both Łos's theorem and the Keisler-Shelah theorem have analogues in a continuous logic setting. However, as continuous logic is *positive*, we get some new features when building ultraproducts. E.g., the formulas we wish to hold in our ultraproduct model need not be true in any of the constituent models.

In this talk I will explore some of the possibilities ultraproducts offer as a means for approximation. In physics there is a tradition of approximating quantum mechanical systems via finite dimensional spaces, and as an example I will sketch how the properties of metric ultraproducts can be used to justify such approximations. This part of the talk is based on joint work with Tapani Hyttinen.

▸ TAKAYUKI KIHARA, *Recent advances in research on subtoposes of the effective topos*.
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A recent discovery by the speaker is that any subtopos of the effective topos is presented by a bilayer of an oracle and a majority notion. (1) As tools for controlling an oracle, there is Weihrauch reducibility, and (2) as tools for controlling a majority notion, there are the Rudin-Keisler order and the Katětov order.

1. The key idea underlying the first direction is that an oracle may be viewed as a "site" for a subtopos. The speakers have obtained several applications of this idea to constructive reverse mathematics, such as constructing toposes that separate several semiconstructive principles (as a comment for those interested, we have

used a priority argument to construct some of such toposes!). Recently, we have extended this method to the topos for total computability (the Grayson topos), enabling the separation of principles weaker than Markov's principle.

2. The second direction reveals that, in addition to oracle computability, there is another approach to enhancing computability: Computability by majority. Our recent developments show that the geometric inclusion order on subtoposes (Lawvere-Tierney topologies) obtained from subobjects of double negation sheaves in the effective topos corresponds to a game-theoretic variant (iterated Fubini product) of the Katětov order on lower sets.

In this talk, I will provide an overview of these new discoveries regarding subtoposes of effective topos made by the speaker and collaborators over the past few years.

This series of studies includes collaborative research with Akihito Kajikawa, Masamori Kaku, Keng Meng Ng, Ming Ng, and Satoshi Nakata.

▸ NICHOLAS PISCHKE, *Extracting uniform bounds from proofs in probability theory.*
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Probability theory, at the surface, seems to require the use of proof-theoretically strong principles even to develop some of the most basic notions. Contrary to these perceived logical limitations, a systematic approach for extending the program of proof mining to this area has recently been proposed (cf. [2] as well as [1]), fundamentally based on the use of probability contents (i.e., finitely additive $[0, 1]$-valued measures defined on algebras of sets).

In this talk, I provide a high-level overview of the main ideas of this endeavor. In particular, I discuss how one can define extended systems of finite-type arithmetic tailored to probability contents, and how the outer measure can be used to provide a structured approach towards recognizing complex probabilistic statements formally in the language of the underlying theory. Combined with proof-theoretic tools like Gödel's functional interpretation and Howard's majorizability, these systems and translations give rise to new probabilistic logical metatheorems which allow for the extraction of computable and highly uniform bounds from (non-effective) proofs of probabilistic existence statements. Moreover, Kohlenbach's uniform boundedness principle in that context allows for a tame access to continuity properties of the measure and facilitates an elimination of the principle of σ-additivity from proofs.

These metatheorems provide a logical explanation for the success of previous proof mining case studies, and in particular the various striking uniformities already observed in practice, and continue to prove themselves to be widely effective. I end by giving a brief overview of some of the central recent case studies, and indicating directions of current and future applied work stemming from these logical results.

[1] M. Neri, P. Oliva and N. Pischke, *A systematic way of analysing proofs in probability theory*, arXiv:2604.08078 (2026).

[2] M. Neri and N. Pischke, *Proof mining and probability theory*, ***Forum of Mathematics, Sigma***, vol. 13 (2025), no. e187.

Abstracts of invited talks in the Special Session on Computability in Analysis

▸ MANON BLANC, *Complexity in computable analysis: the notions of costs for analogue models.*

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In computer science, discrete, or digital, structures have been widely studied. It comes from highly discrete settings of today's computers: they represent and compute over finite quantities. Their physical components are constrained to operate in a binary true/false regime. Thus, the mathematical models of computation that we use to describe them, such as Turing machines, are discrete: they operate with integers and/or words on a finite alphabet, step by step, that is, in discrete time. However, the world around us is continuous and can be described by ordinary differential equations (ODEs), like in physics and other applied sciences, such as biology. For example, the computers we currently use are made up of analogue electronic components, which are naturally defined, in physics, by certain ODEs over continuous settings. We also have mathematical models for analogue machines and analogue computers, such as the "General Purpose Analog Computer" proposed by Claude Shannon in the 1940s ([5]). These models can be seen as the analogue equivalent of Turing machines.

Now, we have quite good characterisations of the notions of cost, or complexity, in discrete models, which are all equivalent up to some polynomial thanks to the Church-Turing thesis. We are interested in having good characterisations of complexity for analogue and continuous models. Also, we study the computing power of such models: understanding whether we could compute more and faster with them. We present here some existing results ([1, 2, 3, 4]) regarding (ODE-based) characterisations of time and space for continuous problems in the model of computable analysis.

[1] Manon Blanc, ***Discrete-Time and Continuous-Time Systems over the Reals: Relating Complexity with Robustness, Length and Precision***, *Institut Polytechnique de Paris*, PhD Thesis, 2025.

[2] Olivier Bournez and Amaury Pouly, ***A survey on analog models of computation***, *Handbook of Computability and Complexity in Analysis*, Theory and Applications of Computability, Springer, 2021.

[3] Riccardo Gozzi, ***Analog characterization of complexity classes***, *Universidade de Lisboa Instituto Superior Tecnico*, PhD Thesis, 2022.

[4] Amaury Pouly, ***Continuous models of computation: from computability to complexity***, *Institut Polytechnique de Paris*, PhD Thesis 2015.

[5] Claude E. Shannon, ***Mathematical Theory of the Differential Analyser***, *Journal of Mathematics and Physics MIT*, vol. 20 (1941), pp. 337–354.

► MATEA ČELAR, *Computable subcontinua and effective decomposability of arcs and arc-like continua.*
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A continuum (compact and connected metric space) is said to be *decomposable* if it can be written as a union of two proper subcontinua. Although it may seem that all nontrivial continua are decomposable, indecomposable continua naturally arise in various contexts in dynamical systems [1]. In computable topology, decomposability arises as a condition in the investigation of computable subcontinua of semicomputable chaniable continua [3].

We aim to investigate which computable metric continuua are expressible as a union of two computable proper subcontinua. We will call such continua *effectively decomposable*. A decomposable continuum can alternatively be characterised as a continuum which contains a proper subcontinuum with non-empty interior [4]. Therefore, determining whether a given continuum is effectively decomposable is closely related to identifying its computable proper subcontinua.

In this talk, we will examine effectively compact continua which contain (a homeomorphic copy of) $\mathbb{R}$ as an open subset and, more generally, (topologically) decomposable chainable (*arc-like*) continua. We will show how to construct computable proper subcontinua of such continua and how this implies that such continua are effectively decomposable.

This talk contains joint work with V. Čačić, M. Horvat, Z. Iljazović, M. Jelić and D. Tarandek.

[1] M. Barge and R.M. Gilette, *Indecomposability and dynamics of invariant plane separating continua*, ***Contemporary Mathematics***, vol. 117 (1991), pp. 13–38.

[2] V. Čačić, M. Čelar, M. Horvat, and Z. Iljazović, *Computable approximations of semicomputable graphs*, ***Logical Methods in Computer Science***, vol. 22 (2026).

[3] Z. Iljazović and B. Pažek, *Computable intersection points*, ***Computability***, vol. 7 (2018), pp. 57–99.

[4] A. Illanes, ***Continuum Theory***, Universitext, Springer Cham, 2025.

▸ HOLGER THIES, *Extracting certified programs from type-theoretic proofs in analysis and topology.*
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Recent developments in proof assistants based on dependent type theory have made it possible to formalize substantial parts of analysis and topology while preserving computational content. In this talk, we discuss approaches to extracting certified programs from formal proofs in constructive type theory, with a focus on computable analysis and exact real computation. Topics include efficient representations for exact real computation, operations on hyperspaces and subsets, and solution operators for ordinary differential equations.

Parts of the talk are based on joint work with Michal Konecný and Sewon Park.

Abstracts of invited talks in the Special Session on History of Logic and Computing

▸ MATHILDE FICHEN, *Anatomy of a programming language: Prolog and the beginnings of European artificial intelligence research.*
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The programming language Prolog, developed in the early 1970s by research teams in Marseille and Edinburgh, highlights the difficulty of grasping the epistemological nature of the complex object that a programming language represents. Whereas most pre-existing programming languages belong to a common genealogy - a language being created either as a continuation of, or in reaction to, a pre-existing language deemed insufficient - thereby granting these objects membership within the same technical family, Prolog was not initially designed as a programming language. The language emerged through successive iterations from an automated theorem-proving tool used within a natural language processing project. This gradual emergence also makes it difficult to pinpoint the exact moment when Prolog “became” a programming language and ceased to be merely a proving tool. Prolog's particular genealogy calls into question, from within, the boundaries defining membership in the category of technical objects. By studying the social, scientific, and cultural context in which Prolog first appeared and subsequently developed - namely, the emerging field of artificial intelligence research in Europe during the 1970s and 1980s - we seek to better characterize the nature of this complex object by observing the concrete manifestations of the language's existence. Particular attention will be paid to the performative power of the language, which helped disseminate the mechanisms of automated deduction that were then predominant in AI research during those decades.

▸ GIUSEPPE PRIMIERO, *On verifying computational systems: from functional to non-functional properties.*
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The history of formal verification traces an evolving understanding of computational correctness. Early approaches treated programs as abstract and fully specified artefacts, defining correctness as input–output conformity grounded in mathematical proof. With concurrency and large-scale systems, verification expanded to address non-determinism, introducing temporal and probabilistic notions of correctness. In the era of machine learning and AI, systems are opaque, adaptive, and embedded in sociotechnical contexts: correctness now extends beyond functional behaviour to non-functional properties such as trustworthiness, fairness, transparency, and responsibility. We overview this trajectory to show that the central question of formal verification is no longer only whether a system computes correctly, but whether its behaviour can be formally shown to be trustworthy and responsibly governed.

▸ JOHN V. TUCKER, *Writing the history of logic and computing.*
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What has mathematical and philosophical logic done for the development of computing? Surely a great deal? Isn't logic a foundation of all computing? I will explore historically these broad questions with software development in mind, especially programming language design, specifying and reasoning about programs and systems, and processing natural languages. What are insights that might stand the ruthless tests of technological fashion, cultural memory and economics of education?

Abstracts of invited talks in the Special Session on Model Theory

▸ SEAN COX, JONATHAN FEIGERT, MARK KAMSMA*, MARCOS MAZARI-ARMIDA, AND JIŘÍ ROSICKÝ, *Cofibrant generation of pure monomorphisms in presheaf categories.*
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The title of this talk, and its main result, are purely category-theoretic. However, we use model-theoretic methods to obtain the result. For a fixed monoid S, there is an algebraic question whether or not pure monomorphisms between sets with an S-action are cofibrantly generated. In (positive) model theory, we sometimes call pure monomorphisms immersions: those homomorphisms that reflect solutions to systems of equations. An earlier result by Lieberman, Vasey and Rosický established an equivalence between the existence of a stable independence relation on a category and cofibrant generation of a certain class of morphisms. We use this equivalence, as well as ideas of Mustafin, to characterise for which monoids S the class of pure monomorphisms is cofibrantly generated: those such that for every $a, b \in S$ there is $c \in S$ with $a = cb$ or $b = ca$. Our methods directly go through in the greater generality of presheaf categories, hence the title, and main result, of the talk.

This is joint work with Sean Cox, Jonathan Feigert, Marcos Mazari-Armida and Jiří Rosický.

▶ ANNA DE MASE, *Automatic definability of valuations from an order-topological perspective.*
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An ordered abelian group G (resp. a field k) has automatic ($\emptyset$)-definability if, in every henselian valued field with value group G (resp. residue field k), the valuation is ($\emptyset$)-definable in the language of rings. In joint work with B. Boissonneau, F. Jahnke, and P. Touchard, we characterize such groups via the model-theoretic properties of weak and strong augmentability by infinitesimals, and such fields via t-henselianity, in characteristic 0. In this talk, I show that strong augmentability admits an order-topological formulation, and I discuss analogous questions for weak augmentability. I also present an order-topological formulation of t-henselianity in the setting of orderable fields, building on work of Prestel and Ziegler from 1978. The talk includes results from this joint work, as well as ongoing work with L. S. Krapp and S. Kuhlmann.

[1] Blaise Boissonneau, Anna De Mase, Franziska Jahnke and Pierre Touchard, *Growing spines: ad infinitum et ad infinitesimalia*, arXiv:2512.04932 (2025).

▶ STEFAN MARIAN LUDWIG, *Generalisations of Ax's theorem on pseudofinite fields.*
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A pseudofinite field is an infinite model of the common theory of all finite fields in the language of rings. In 1968, James Ax, building on number-theoretic results, gave an axiomatization of the theory and proved its decidability, as well as a quantifier reduction. Hrushovski generalised this to the algebraic closures of finite fields equipped with a symbol for the Frobenius, and, more recently, to finite fields equipped with an additive character. Motivated by natural number-theoretic examples, we will present a further generalisation of the latter, and, if time permits, consider the theory arising from the combination of both of Hrushovski's works.

Abstracts of invited talks in the Special Session on Pure and Applied Proof Theory

▶ LEV BEKLEMISHEV, *Automatic structures and the problem of natural well-orderings*.
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We explore the idea of using automatic and other very efficient presentations of structures to deal with the conceptual problem of natural proof- theoretic ordinal notations. We conclude that this approach still does not meet the goals.

This is joint work with Fedor N. Pakhomov.

▶ PAULO FIRMINO, *Quantitative results on a generalized viscosity approximation method*.
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In this talk we present the results of a quantitative analysis of the asymptotic behaviour of the generalized Viscosity Approximation Method (genVAM), an iteration we studied in joint work with Leuştean. The genVAM iteration is a generalization of VAM, an iteration studied in Banach spaces by Xu et al. [7], to W-hyperbolic spaces and to families $(T_n)_{n\in\mathbb{N}}$ of mappings satisfying certain resolvent-like conditions (introduced by Leuştean, Nicolae and Sipoş [5] in their abstract analysis of the Proximal Point Algorithm). The genVAM iteration also generalizes the abstract HPPA studied by Aoyama, Kimura, Takahashi, and Toyoda [1], itself a generalization of the well-known Halpern-type Proximal Point Algorithm. We computed rates of $((T_n)$-)asymptotic regularity, as well as rates of T_m-asymptotic regularity for all $m \in \mathbb{N}$. In this way, we extend to this very general setting the quantitative asymptotic regularity results we had obtained for the VAM iteration in Banach spaces [2]. Furthermore, as an application of a lemma due to Sabach and Shtern [6], linear such rates are computed for particular choices of the parameter sequences. On the setting of complete CAT(0) spaces, we compute rates of metastability for the abstract HPPA. We then, relying on results by Kohlenbach and Pinto [4], compute rates of metastability for genVAM, in doing so establishing a qualitative convergence result for the iteration. This talk reports on [3], recent joint work with Leuştean.

[1] K. Aoyama, Y. Kimura, W. Takahashi, and M. Toyoda, *Approximation of common fixed points of a countable family of nonexpansive mappings in a Banach space*, ***Nonlinear Analysis***, vol. 67 (2007), pp. 2350–2360.

[2] P. Firmino and L. Leuştean, *Quantitative asymptotic regularity of the VAM iteration with error terms for m-accretive operators in Banach spaces*, ***Zeitschrift für Analysis und ihre Anwendugen***, vol. 44 (2025), pp. 501–519.

[3] ——— *Quantitative results on a generalized viscosity approximation method*, submitted, arXiv:2512.09968 [math.OC] (2025)

[4] U. Kohlenbach and P. Pinto, *Quantitative translations for viscosity approximation methods in hyperbolic spaces*, ***Journal of Mathematical Analysis and Applications***, vol. 507 (2022), no. 2, 125823.

[5] L. Leuştean, A. Nicolae, and A. Sipoş, *An abstract proximal point algorithm*, ***Journal of Global Optimization***, vol. 72 (2018), pp. 553–577.

[6] S. Sabach and S. Shtern, *A first order method for solving convex bilevel optimization problems*, ***SIAM Journal on Optimization***, vol. 27 (2017), pp. 640–660.

[7] H.-K. Xu, N. Altwaijry, I. Alughaibi, and S. Chebbi, *The viscosity approximation method for accretive operators in Banach spaces*, ***Journal of Nonlinear and Variational Analysis***, vol. 6 (2022), no. 1, pp. 37–50.

Abstracts of invited talks in the Special Session on Set Theory

▸ SUMUN IYER, *The Knaster continuum homeomorphism group.*
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A *Knaster continuum* is one which can be written as an inverse limit of intervals $[0,1]$ where each bonding map is a continuous open surjection mapping 0 to 0. Knaster continua are examples of *indecomposable* continua, they cannot be written as the union of two proper subcontinua. The *universal* Knaster continuum, K, is a Knaster continuum which continuously surjects onto any Knaster continuum. The group $\mathrm{Homeo}(K)$ of homeomorphisms of K is a non-locally compact Polish group. The main theorem is that $\mathrm{Homeo}(K)$ contains an open subgroup with a comeagre conjugacy class.

▸ BORIŠA KUZELJEVIĆ, *A dichotomy for transitive lists.*
Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad, Trg Dositeja Obradovića 4, 21000 Novi Sad, Serbia.
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This talk presents a joint work with Roy Shalev and Stevo Todorčević. We will formulate, for any regular cardinal λ, *a transitive list dichotomy* $\mathrm{LD}(\lambda^+,\lambda)$ and show that $\mathrm{LD}(\omega_2,\omega_1)$ is consistent with CH assuming the existence of a weakly compact cardinal, while $\mathrm{LD}(\omega_1,\omega)$ is a consequence of $\mathrm{MA}_{\aleph_1}$. This dichotomy is a direct generalization of the statement that every λ^+-Aronszajn tree is special, and it implies, for example: that every λ^+-tower in $(\mathcal{P}(\lambda),\subseteq^*)$ is Hausdorff, the nonexistence of λ^+-Souslin lower semilattices, the nonexistence of certain strongly unbounded colorings (assuming $\lambda^{<\lambda}=\lambda$), and the nonexistence of (λ^+,λ^+)-T-gaps in $\mathcal{P}(\lambda)$.

▸ ALEJANDRO POVEDA, *The directedness of the Rudin–Keisler order on measurable cardinals.*
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There is a long-standing project aimed at classifying ultrafilters. One of the most successful tools in this endeavor has been the Rudin-Keisler order $\leq_{\mathrm{RK}}$. A central feature of the Rudin-Keisler order is that it provides a framework for understanding the intrinsic structure of an ultrafilter through its relative position within the order. For instance, selective (a.k.a. Ramsey) ultrafilters are those that are $\leq_{\mathrm{RK}}$-minimal. In this talk we will focus on the directedness of $\leq_{\mathrm{RK}}$ on the set of κ-complete ultrafilters on κ, $\mathfrak{U}_\kappa$. Classical theorems of Kunen and, independently, Comfort–Negrepontis deduce the directedness of $\leq_{\mathrm{RK}}$ from instances of compactness; for instance, if κ is κ-compact then $\langle\mathfrak{U}_\kappa,\leq_{\mathrm{RK}}\rangle$ is $(2^\kappa)^+$-directed. On the other hand, it can be showed that, if there is no inner model for a Woodin cardinal, $\langle\mathfrak{U}_\kappa,\leq_{\mathrm{RK}}\rangle$ fails to be σ-directed whenever κ is the first strong cardinal κ in the core model.

In light of these, a natural conjecture is that the directedness of the Rudin-Keisler order at measurable cardinals can only be produced from large cardinals in the realm of κ-compactness or strong compactness. In joint work with Yair Hayut, we refute this by proving that the κ^+-directedness of $\leq_{\mathrm{RK}}$ over κ-complete ultrafilters is equiconsistent with the existence of a cardinal κ with Mitchell order κ. This is obtained after bridging the directedness of $\leq_{\mathrm{RK}}$ with a new principle (The Gluing Property) and showing that the latter can be forced, from optimal assumptions, using nonstationary support iterations of Prikry-type forcings.

Absracts of invited talks in the Special Session on Type Theory

▸ LOÏC PUJET, *Revisiting Martin Hofmann's setoid model.*
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In his seminal PhD thesis [1], Martin Hofmann proposed two definitions of the setoid model of type theory in the language of intensional type theory itself, in order to recover extensionality principles such as function extensionality, proposition extensionality and quotient types. Unfortunately, due to the lack of extensionality, his models only validate some of the rules of type theory up to setoid equality.

In this talk, I will present a variation on Hofmann's $\mathbf{S}_1$ model which uses strict propositions to sidestep the lack of extensionality. It supports all of the constructors of type theory as well as the desired extensionality principles, without sacrificing the computational content of equality proofs. I will discuss the consequences of this model on constructive fragments of the axiom of choice, proof erasure, and impredicativity. I will also draw parallels with models of univalent type theory in cubical and semi-cubical sets.

[1] Martin Hofmann, *Extensional concepts in intensional type theory*, University of Edinburgh, PhD Dissertation, 1995.

▸ ZHIXUAN YANG, *Functional data structures in monoidal categories.*
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An achievement of functional programming is the invention of many efficient purely functional data structures and algorithms. Motivated by the observation that many of these data structures and algorithms can be typed in linear type systems, in this talk I will show my ongoing work of studying functional data structures at the generality of monoidal categories. This generalisation unlocks the door to a strange new world of data structures: lists and free monoids no longer need to be isomorphic in non-closed monoidal categories; cons-lists and snoc-lists no longer need to coincide in non-symmetric monoidal categories; the time complexities of accessing the two components of a monoidal product may be different; and pattern matching may only be possible for the first variable of a context, but not the others, in categories where the monoidal product only distributes from the right but not the left.

As an application of this generalisation, I will show how Chris Okasaki's *catenable lists* [1] can be typed in a certain non-commutative linear type system and interpreted in the category of endofunctors. This gives us a data structure of abstract syntax trees that supports amortised constant-time-per-variable substitution, eliminating the cost of traversing the inner nodes of the syntax tree, which might be of interest for efficient implementations of type theories and programming languages.

[1] Chris Okasaki, ***Purely Functional Data Structures***, Cambridge University Press, 1998.

Abstracts of contributed talks

▶ AIZHAN ALTAYEVA* AND BEIBUT KULPESHOV, *On algebras of binary formulas in circularly ordered structures: piecewise monotonic case.*
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Algebras of binary isolating formulas [4, 3] are a tool for describing relationships between elements of the sets of realizations of a type at the binary level. The notion of *weak circular minimality* was studied initially in [2].

THEOREM (piecewise monotonic case, [1]). *Let M be an $\aleph_0$–categorical 1-transitive non-primitive weakly circularly minimal structure of convexity rank greater than 1, $dcl(\{a\}) = \{a\}$ for some $a \in M$. Suppose that there exists a convex-to-right formula $R(x, y)$ such that $r(y) := \text{rend } R(M, y)$ is piecewise monotonic-to-left on M. Then M is isomorphic up to binarity to $M'_{s,m,k} := \langle M, K^3, E_1^2, E_2^2, \dots, E_s^2, E_{s+1}^2, R^2\rangle$, where M is a circularly ordered structure, densely ordered, $s \geq 1$; E_{s+1} partitions M into m infinite convex classes; E_i for $1 \leq i \leq s$ partitions each E_{i+1}-class into infinitely many infinite convex E_i-subclasses; $R(M, a)$ has no right endpoint and $r^k(a) = a$ for all $a \in M$ and some $k \geq 2$, where $r^k(y) := r(r^{k-1}(y))$; for every $1 \leq i \leq s+1$ and any $a \in M$*

$$M'_{s,m,k} \models \neg E_i^*(a, r(a)) \land \forall y\big(E_i(y, a) \to \exists u[E_i^*(u, r(a)) \land E_i^*(u, r(y))]\big),$$

$m \geq 4$, k is even and divides m; r is monotonic-to-left on every E_{s+1}-class and monotonic-to-right on M/E_{s+1}.

THEOREM. *The algebra $\mathfrak{P}_{M'_{s,4,4}}$ of binary isolating formulas with piecewise monotonic-to-left function r has $8s + 9$ labels, is non-commutative and strictly $(2s + 3)$-deterministic for every $s \geq 1$.*

This research has been funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP22685890).

[1] B.Sh. Kulpeshov, *Definable functions in the $\aleph_0$-categorical weakly circularly minimal structures*, ***Siberian Mathematical Journal***, Vol. 50, No. 2 (2009), pp. 282–301.

[2] B.Sh. Kulpeshov and H.D. Macpherson, *Minimality conditions on circularly ordered structures*, ***Mathematical Logic Quarterly***, Vol. 51, No. 4 (2005), pp. 377–399.

[3] I.V. Shulepov and S.V. Sudoplatov, *Algebras of distributions for isolating formulas of a complete theory*, ***Siberian Electronic Mathematical Reports***, Vol. 14 (2014), pp. 380–407.

[4] S.V. Sudoplatov, *Classification of countable models of complete theories*, Novosibirsk: Novosibirsk State Technical University, 2018.

▶ DAVID APPADOURAI, *Localising overgeneration: a three-strata decomposition of formal proof.*
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Tanswell [2] argues that derivationist accounts of mathematical proof face an overgeneration problem: informal proofs can be formalised in multiple, structurally non-equivalent ways, and this multiplicity undermines the philosophical significance of the proof–formalisation correspondence.

Tactic-based proof assistants like Lean and Coq make visible a three-strata decomposition that localises overgeneration. The strata are: (1) the informal proof (argumentative prose), (2) the tactic script (executable code that instructs the elaborator to construct a proof term), and (3) the proof term (a typed expression in the Calculus of Inductive Constructions verified by the kernel).

The transition from stratum 1 to stratum 2 introduces the multiplicity: one informal proof admits multiple tactic scripts, each reflecting different authorial choices about decomposition and generalisation. But the transition from stratum 2 to stratum 3 is deterministic relative to a fixed environment (Lean version, library commit, import graph): a given tactic script yields exactly one proof term. The elaborator is non-originating in a precise sense: unification and instance resolution work only on content already in the script or the environment.

This partition, developed through a Lean formalisation of IMO 2024 Problem 6, is an epistemological result: derivationist accounts must answer the selection question at the formaliser's authored choices, not at the elaborator's mechanical construction. The partition is visible only when the formal pole of the standard view [1] is decomposed into script and term.

[1] Y. Hamami, *Mathematical rigor and proof*, ***The Review of Symbolic Logic***, vol. 15 (2022), no. 2, pp. 409–449.

[2] F. Tanswell, *A problem with the dependence of informal proofs on formal proofs*, ***Philosophia Mathematica***, vol. 23 (2015), no. 3, pp. 295–310.

► ZIBA ASSADI, *Fuzzy Semantics for Normative Reasoning*.
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(This is a joint work with Paola Inverardi). AI systems are increasingly integrated into daily life, raising concerns about ethical, social, and legal compliance. To be trustworthy, such systems must operate according to well-defined norms, including obligations, permissions, and prohibitions. Translating these norms into a computable form remains challenging, as classical binary logic cannot capture the gradability, context-dependence, and uncertainty inherent in real-world normative reasoning. We propose a fuzzy-logical framework for representing and reasoning about norms, extending classical deontic approaches to accommodate partial compliance and graded satisfaction. Building on the SLEEC (Social, Legal, Ethical, Empathetic, and Cultural) rules, we formalize norms as entities whose fulfillment is quantified along a continuum rather than as binary true/false values. This enables nuanced reasoning about obligation strength, violation severity, and the prioritization of conflicting norms. Our methodology replaces the "unless" structures of traditional SLEEC formalizations with explicit IF–THEN–ELSE constructs augmented with fuzzy truth values. This ensures deterministic evaluation, eliminates semantic ambiguity, and produces directly executable decision models. Fuzzy reasoning is applied through four steps: (1) designation, identifying explicit and implicit concepts; (2) fuzzification and aggregation, computing the degree to which conditions are satisfied; (3) compilation, structuring and validating the fuzzy output into IF–THEN–ELSE rules; and (4) defuzzification, producing a final quantified decision outcome. We illustrate the approach with a case study in which an AI system balances privacy versus assistance, dynamically estimating user distress and making ethically justified decisions under conflicting norms. By quantifying partial compliance and resolving soft conflicts, our framework enables AI systems to manage normative dilemmas under uncertainty. Overall, this work bridges formal normative reasoning and fuzzy logic, offering a rigorous yet flexible foundation for AI systems capable of graded, context-aware, and ethically informed decision-making.

[1] Ziba Assadi and Paola Inverardi, *Embedding Normative Requirements in Fuzzy Logic*, ***Requirements Engineering: Foundation for Software Quality*** (Poznań, Poland), (Renata Guizzardi and João Araújo, editors), Lecture Notes in Computer Science, vol. 16497, Springer Cham, 2026, pp. 219–229.

[2] ——— *Fuzzy Representation of Norms*, arXiv:2601.04249 (2026).

► RAMAZAN AYUPOV, *Aanalyticity and contemporary formal theories: Hintikka's typology and the case of type theory.*
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This talk offers a critique of Jaakko Hintikka's analysis of analyticity [2], from the perspective of Martin-Löf type theory. Hintikka distinguishes four types of analyticity: conceptual truth; subformulaic or structural analyticity; analyticity as reasoning that introduces no new entities; and analyticity as either tautological or informationally empty. Hintikka's aim is to formalise a Kantian insight about mathematics: mathematical reasoning is synthetic because it requires construction and cannot be reduced to mere conceptual analysis. However, Hintikka fails to preserve Kantian constructive dimension of analyticity; it is this dimension that will be realized by Martin-Löf [3].

I argue that this Hintikka framework is insufficient for contemporary formal theories. Particularly Martin-Löf's intuitionistic type theory and its subsequent reinterpretations. In MLTT, the analytic-synthetic distinction is formulated at the level of judgments rather than propositions or argument steps. This shift reveals forms of analyticity tied to proof, construction, hypothetical reasoning and, in later discussions, evaluation and computational semantics [1]. For this reason, MLTT cannot simply be fitted into Hintikka's typology.

[1] Bruno Bentzen, *Analyticity and Syntheticity in Type Theory Revisited*, ***The Review of Symbolic Logic***, vol. 7 (2024), no. 4, pp. 1119–1145.

[2] Jaakko Hintikka, *An Analysis of Analylicity*, ***Logic, Language-Games and Information: Kantian Themes in the Philosophy of Logic*** Oxford: Clarendon Press, 1973, pp. 123–149.

[3] Per Martin-Löf, *Analytic and Synthetic Judgements in Type Theory*, ***Kant and Contemporary Epistemology*** (Parrini, P., editor), The University of Western Ontario Series in Philosophy of Science, vol. 54, Dordrecht: Springer, 1994, pp. 87–100.

► JEREMY BEARD, *Limit models in strictly stable AECs.*
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Limit models are a useful replacement for saturated models in abstract elementary classes, where saturated models can be less well behaved than in first order. The question of which limit models are isomorphic has proved important when approaching the main test question of AECs, Shelah's categoricity conjecture. Historically, limit models have mostly been studied in superstable AECs.

In this talk, we'll discuss recent results that shed light on how limit models behave in *strictly stable* AECs. In particular, we present (as much as time allows):

1. A full characterisation of isomorphism types of limit models with a nicely behaved forking-like relation - 'long' limits are isomorphic, and 'short' limits are non-isomorphic. In particular, this applies to all first order stable theories [3].
2. The most general 'positive' isomorphism of limit models result (to my knowledge), assuming only a relation with weak forms of uniqueness and extension (similar to

λ-splitting) [1].

3. 'Long' limit models are disjoint (non-forking) amalgamation bases [2].

[1] Jeremy Beard, *Long limit models are isomorphic assuming a splitting-like relation*, arXiv.2511.18665 (2025).

[2] ——— *Disjoint non-forking amalgamation in stable AECs*, arXiv.2601.12439 (2026).

[3] Jeremy Beard and Marcos Mazari-Armida, *On the spectrum of limit models*, ***Annals of Pure and Applied Logic***, vol. 176, no. 10, pp. 103647.

▸ MARIJA BORIČIĆ JOKSIMOVIĆ, *On preprobability natural deduction.*
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The system of preprobabilistic inference rules, denoted by $\mathbf{NK}\pi$, similarly as in Boričić [1] and [2], from Gentzen's **NK**, for natural deduction for classical logic (see Gentzen [4], Prawitz [8]), and π — for 'preprobability', consists of inference rules that allow the introduction, (I-), and elimination, (E-), for any logical connective. Each propositional formula A has its preprobabilistic counterpart A^r, for $r \in \mathbf{B}$, where $\langle \mathbf{B}, ...\rangle$ is a finite Boolean algebra, with intended meaning that 'preprobability of A is greater than or equal to r'.

The only axiom of $\mathbf{NK}\pi$ is A^1, for each classically provable proposition A, and, for instance, the inference rules dealing with conjunction look as follows:

$$\frac{A^r \quad B^s}{(A\wedge B)^{\min(r,s)}}(I\wedge) \qquad \frac{(A\wedge B)^r}{A^r}, \frac{(A\wedge B)^r}{B^r}(E\wedge)$$

Models for systems of probability logic are based on additivity condition. This condition is, in our opinion, incompatible with the elementary expectations and requirements of proof theory. In order to obtain a sound and complete system that is proof theoretically acceptable, and not far from the concept of probability, we propose a weaker semantics than usual. Namely, instead of the condition of additivity, we adopt a weaker monotonicity condition, which promises the possibility of some normalization theorem for the system of natural deductions.

Our model, in spirit of Carnap–Popper–Hailperin–Leblanc approach, [3], [5], [6], and [7], is defined as follows. Let For be the set of all propositional formulae. Then a mapping $\pi : \mathrm{For} \to \mathbf{B}$ will be an $\mathbf{NK}\pi$—*model* (or, simply, *model*), if it satisfies the following conditions:

(a) $\pi(\top) = 1$;

(b) if $A \to B$ in classical logic, then $\pi(A) \leq \pi(B)$ (monotonicity), and

(c) $\pi(\neg A) = \mathbf{c}(\pi(A))$, where $\mathbf{c}$ is the complementation operation in $\langle \mathbf{B}, ...\rangle$.

Our semantical goal is to obtain both, a sound and complete system, and a normalizable system from a syntactic point of view.

[1] M. Boričić, *Inference rules for probability logic*, ***Publications de l'Institut Mathematique***, 100(114), 2016, pp. 77–86.

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[3] R. Carnap, ***Logical Foundations of Probability***, University of Chicago Press, Chicago, 1950.

[4] G. Gentzen, *Untersuchungen über das logische Schliessen*, ***Mathematische Zeitschrift***, vol. 39 (1934–35), pp. 176–210, 405–431, or G. Gentzen, ***Collected Papers***, (ed. M. E. Szabo), North–Holland, Amsterdam, 1969.

[5] T. Hailperin, *Probability logic*, ***Notre Dame Journal of Formal Logic***, vol. 25 (1984), pp. 198–212.

[6] H. Leblanc, *Probability functions and their assumption sets — the singular case*, ***Journal of Philosophical Logic***, vol. 12 (1983), pp. 382–402.

[7] K.R. Popper, *Two autonomous axiom systems for the calculus of probabilities*, ***The British Journal for the Philosophy of Science***, vol. 6 (1955), pp. 51–57, 176, 351.

[8] D. Prawitz, ***Natural Deduction. A Proof–theoretical Study***, Almquist and Wiksell, Stockholm, 1965.

▸ HARRY BRYANT*, ANDREW LAWRENCE, MONIKA SEISENBERGER, AND ANTON SETZER, *Certified assurance for railway interlockings: From SAT proofs to mechanised verification in Rocq and Agda.*
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Safety-critical railway interlockings demand the highest levels of assurance, making formally verified proof checking essential. We present an approach for certifying Z3-generated SAT proof logs of unsatisfiability using Rocq and Agda, focusing on correctness guarantees and extracting a verified proof checker. In Rocq, we implement a checker for RUP proofs of CNF unsatisfiability, including Tseitin transformation steps, reduced Tseitin steps, and deletion steps to improve efficiency. We also developed an Agda prototype formalising Tseitin transformations and RUP inferences to reconstruct Z3 proof-logs. Correctness is established by proving inferences preserve logical entailment: any model of the assumptions satisfies all derived conclusions. Deriving falsity certifies the assumptions are unsatisfiable, enabling integration of Z3 proofs without modifying the proof assistants. Our Rocq framework offers an extensible methodology for certified proof checking: (1) implement the checking procedure, (2) prove soundness of each inference step, and (3) extract a verified OCaml checker. For scalability, we introduce proof-design patterns, including a tautology-based technique for validating Tseitin steps. While prior work, such as SMTCoq and CoqHammer, demonstrates how results from SMT solvers can be formally verified within interactive theorem provers, we extend this to Z3 and its recently introduced RUP format. However, the methodology for creating correctness proofs is general and applicable to other SMT theories. Verification is crucial to railway interlockings, where ensuring logical soundness and safety are essential.

▸ GABRIELE BURIOLA*, GIACOMO COZZI, ANTONIA DAMMRAU, AND ANTON FREUND, *Well-ordering principles over recursive comprehension.*
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Ordinal analysis and well-ordering principles [4] play a crucial role in proof theory. Recently, Pakhomov and Walsh [3, Lemma 3.8] have proved, over ACA_0, an equivalence result allowing to convert well-ordering axioms ($\forall X\,[\mathsf{WO}(X) \to \psi(X)]$) to well-ordering rules $\left(\frac{\mathsf{WO}(X)}{\psi(X)}\right)$; at least for what concerns Π^1_1 formulas.

Since their proof exploits the Π^1_1 completeness of well-orders, which requires ACA_0, in order to obtain a corresponding version over RCA_0, we need a suitable replacement; in particular, we rely on well-founded partial orders and ranking functions. Given two partial orders P, Q, a *ranking* is a function $r : T(P) \to Q$ from the set of finite strictly P-descending sequences to Q such that $\tau \sqsubset \sigma \Rightarrow r(\tau) >_Q r(\sigma)$; if such a ranking exists, we say that P is *ranked* by Q. Crucially, if P is ranked by Q and Q is well-founded, then also P is well-founded. Our result is the following, where PO and WF denote respectively the property of being a partial order and a well-founded partial order:

THEOREM. *Consider an $\mathcal{L}_2$-theory T that extends RCA_0 and can be axiomatized by $\Pi^1_2(\Sigma^0_3)$-sentences. Let $\psi(X)$ be a $\Pi^1_2(\Sigma^0_3)$-formula with*

$$T \vdash \forall X, Y \in \mathsf{PO}\,(\text{“}X \textit{ is ranked by } Y\text{”} \land \psi(Y) \to \psi(X))\,.$$

Then $T + \forall X(X \in \mathsf{WF} \to \psi(X))$ is $\Pi^1_1(\Pi^0_3)$-conservative over T extended with the rule “whenever T proves $P \in \mathsf{WF}$ for a closed comprehension term P, it proves $\psi(P)$”.

With this equivalence at our disposal, we aim to study ordinal analysis over weak theories, such as in [2]. In particular, our goal is to obtain a suitable version over RCA_0 of the following result due to Arai [1, Theorem 3], which eases the computation of the proof-ordinal for a normal ordinal function g:

$$|\mathsf{ACA}_0 + \forall X[\mathsf{WO}(X) \to \mathsf{WO}(g(X))]| = g'(0) = \min\{\alpha > 0 \,|\, \forall \beta < \alpha\ g(\beta) < g(\alpha)\}.$$

[1] T. ARAI, *Proof-theoretic strengths of the well-ordering principles.* ***Archive for Mathematical Logic***, vol. 59 (2020), no. 3–4, pp. 257–275.

[2] L. CARLUCCI, L. MAINARDI, AND M. RATHJEN, *A Note on the Ordinal Analysis of $RCA_0 + WO(\sigma)$*, ***Computability in Europe 2019: Computing with Foresight and Industry*** (Durham, UK), (F. Manea, B. Martin, D. Paulusma and G. Primiero, editors), Lecture Notes in Computer Science, vol. 11558, Springer Cham., 2019, pp. 144-155.

[3] F. PAKHOMOV AND J. WALSH, *Corrigendum to Reducing ω-model reflection to iterated syntactic reflection*, ***Journal of Mathematical Logic***, vol. 23 (2023), no. 3.

[4] M. RATHJEN, *Well-Ordering Principles in Proof Theory and Reverse Mathematics*, ***Axiomatic Thinking II***, (F. Ferreira, R. Kahle, and G. Sommaruga, editors), Springer Cham., 2022, pp. 89–127.

▸ REZAN BURCU BÜYÜKIKIZ, *An introduction to Husserl's phenomenology: on the nature of numbers.*
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This talk examines the conceptual development of Edmund Husserl's philosophy of mathematics, tracing the transition from his 1887 habilitation thesis, *Über den Begriff der Zahl*, to his 1907 lectures, *The Idea of Phenomenology.* We focus on the concept of “Colligation” (*Kolligation*) as a temporal-psychological act of the subject and its eventual transformation into the phenomenological concept of “givenness” (*Gegebenheit*). Drawing on Elisabeth Ströker's transcendental analysis, we argue that Husserl's early work on numbers provides a crucial grounding for pure logic and epistemology. Finally, we discuss how the “Heraclitean flux” of phenomena necessitates a non-anthropocentric inquiry into the invariant structures of mathematical objects within the flow of experience.

▸ LUCA CASTALDO, MATEUSZ ŁEŁYK, AND KONSTANTINOS PAPAFILIPPOU*,

Categoricity-like properties applied to some 'friends' of KF_μ.

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This talk will be about the axiomatic theory of truth over arithmetic KF_μ, first introduced by Burgess [4], as well as theories related to it. KF_μ is the theory whose intended model is the least fixed-point of the Kripke jump operator K and axiomatically it extends KF by asserting its minimality. This could make one expect for the theory to have a unique ω model, however this is not the case due to complexity considerations [6]. In the words of [6], KF_μ is not $\mathbb{N}$-categorical. Not all hope is lost though, as KF_μ satisfies the property of being solid [5] (interpretations of its models do not induce non-trivial cycles with respect to definable isomorphism) which expresses a form of 'internal' categoricity.

This leaves open this in-between space of what the non-standard models of KF_μ look like. In the first part of the talk we will briefly see how one can construct non-standard models of KF_μ. To this end, we will sketch how KF_μ is biinterpretable (in fact, synonymous) with a theory we dub KP_s (expressing that it is the least admissible model of KP). Then we can use tools from admissible set theory [1] (like Barwise completeness and compactness) to build our non-standard models.

On the second part of the talk we will concern ourselves with the interplay between categoricity-like properties (here solidity) and that of the proof theoretic strength of a theory. We define a natural subtheory of KF_μ, whose intended models are the *intrinsic* fixed-points of the Kripke jump operator. Recall that a fixed-point is intrinsic iff it is compatible with every consistent fixed-point. We denote this theory as KF_ι. This theory not solid and lies strictly between $\mathsf{KF} + \mathsf{Cons}$ and KF_μ, however its arithmetical consequences are exactly the same as those of KF_μ. Similarly to KF_μ, it is also not $\mathbb{N}$-categorical, in the sense that there are models of KF_ι that are not intrinsic—which once more comes from complexity considerations using the complexity of the greatest intrinsic fixed-point, computed by Burgess [2].

If time permits, we will also compare the Burgess KF_μ with its counterpart by Cantini in [3], denoted $\mathsf{KF}^+ + \mathsf{GI}$. We show that the latter is provable in the former but leave the problem of their equivalence as an open question.

[1] Jon Barwise, ***Admissible Sets and Structures***, Perspectives in Logic, Cambridge University Press, 2017.

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[3] Andrea Cantini, *Notes on formal theories of truth*, ***Mathematical Logic Quarterly***, vol. 35 (1989), no. 2, pp. 97–130.

[4] John P. Burgess, *Friedman and the axiomatization of Kripke's theory of truth*, ***Foundational adventures: Essays in honor of Harvey M. Friedman*** (Neil Tennant, editor), College Publications, 2014, pp. 125–148.

[5] Ali Enayat, and Mateusz Łełyk, *Categoricity-like Properties in the First Order Realm*, ***Journal for the Philosophy of Mathematics***, vol. 1 (2024), pp. 63–98.

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▸ HORAŢIU CHEVAL, $\mathbb{K}$ *definitions as Matching Logic theories.*
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$\mathbb{K}$ is a language-agnostic verification framework that enables one to formally specify the semantics of programming languages, from which various tools like interpreters, verifiers, and model checkers can be derived automatically. Matching Logic ($\mathbb{ML}$) is a minimal yet expressive logical system intended to serve as the theoretical foundation of $\mathbb{K}$, in the sense that programming language definitions are encoded as $\mathbb{ML}$ theories, and verified program properties are theorems over these theories. In practice, however, this translation happens through a complex compilation process that introduces a significant representational distance between $\mathbb{K}$ definitions and the underlying theory. This is particularly noticeable for the so-called *abstract rewriting rules*, which are one of $\mathbb{K}$'s most important features, and allow one to specify rewrite axioms that apply locally, only to specific subterms of a program's configuration. Currently, these are individually concretized into axioms that do not closely resemble the user-facing syntax.

In this talk, we present a new approach to a denotational semantics of $\mathbb{K}$ in $\mathbb{ML}$ that aims to narrow this representational gap. We will focus on abstract rules, for which we propose a denotation based on a newly developed theory of contexts, which preserves their locality and compositionality.

This is joint work with Xiaohong Chen, Dorel Lucanu and Grigore Roşu.

[1] Xiaohong Chen, Horaţiu Cheval, Dorel Lucanu and Grigore Roşu, $\mathbb{K}$ *definitions as Matching Logic theories, formally*, ***Proceedings of the 29th International Conference on Foundations of Software Science and Computation Structures (FOSSACS '26)*** (Turin, Italy), to appear.

[2] Xiaohong Chen, Dorel Lucanu and Grigore Roşu, *Matching logic explained*, ***Journal of Logical and Algebraic Methods in Programming***, vol. 120 (2021), pp. 100638.

[3] Xiaohong Chen and Grigore Roşu, *Matching μ-logic*, ***Proceedings of the 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS'19)*** (Vancouver, Canada), IEEE, 2019, pp. 1–13.

▸ ALAKH DHRUV CHOPRA, *Well-founded trees with leaf labels, and their applications.*
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We consider the collection $\mathsf{T}_\mathsf{f}^\alpha(Q)$ of well-founded trees of rank at most α with leaf vertices labelled by elements of the quasi-order Q, but restricted to trees that can be finitely represented. The order on Q is extended to an order on $\mathsf{T}_\mathsf{f}^\alpha(Q)$ via (strong) tree homomorphisms. This is a strengthening of the *hyperated finitary powerset operator* $\mathsf{P}_\mathsf{f}^\alpha(Q)$, which is known to preserve the property of being a well-quasi-order. The maximal order type of $\mathsf{P}_\mathsf{f}^\alpha(Q)$ can be bounded using certain *hyperations* of normal ordinal-valued functions, a concept introduced by Fernández-Duque and Joosten [2] and extended to operators by Provenzano [3]. Adapting that proof gives us precise bounds for the maximal order type of $\mathsf{T}_\mathsf{f}^\alpha(Q)$ (as a function of the maximal order type of Q) corresponding almost exactly to hyperations of 2-exponentiation, and thus to the usual Veblen hierarchy of functions defined via ω-exponentiation.

The goal of the talk is to highlight some of their applications to long-standing problems in wqo-theory. In [1], the quasi-order $\mathsf{T_f}(Q)$ – equal to $\mathsf{T_f^\omega}(Q)$ here – is shown to be equivalent to the collection of transfinite sequences over Q with finite range and length less than ω^ω. We can generalize this correspondence, going from $\mathsf{T_f^\alpha}(Q)$ to indecomposable sequences of arbitrary (countable) length ω^α, and use that to state bounds for their maximal order type. In a similar vein, the quasi-order $\mathsf{P_f^\alpha}(Q)$ is equivalent to the collection of hereditarily finite sets $H_f(Q)$ (with urelements Q). Generalizing the rank lets us strengthen the results of [4] by an extension of its proof techniques.

This is a continuation of a talk previously given in the Logic Colloquium 2024.

[1] Alakh Dhruv Chopra and Fedor Pakhomov, *Well-quasi-orders on finite trees and transfinite sequences*, arXiv:2602.09830 (2026).

[2] David Fernández-Duque and Joost J. Joosten, *Hyperations, Veblen progressions and transfinite iteration of ordinal functions*, ***Annals of Pure and Applied Logic***, vol. 164, no. 7–8, pp. 785–801.

[3] Philipp Provenzano, *The reverse mathematical strength of hyperations*, Master's Thesis (2022).

[4] Anton Freund, *On the logical strength of the better quasi order with three elements*, ***Transactions of the American Mathematical Society***, vol. 376, no. 9, pp. 6709–6727.

► NOX COWIE, *Double negation elimination and the multiple succedent structure of LK*.
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The classical natural deduction system NK can be obtained from the intuitionistic NJ by adding a double negation elimination inference rule. On the other hand, the intuitionistic and classical sequent calculi LJ and LK are differentiated by the number of formulae that can be contained in the right-hand side of a sequent. In an LJ-derivation, the right-hand side of a sequent can contain at most one formula; LK is obtained by lifting this restriction. This distinction between LJ and LK is notably different from that between NJ and NK: the first being purely structural and the second involving the addition of a classical inference pattern. We demonstrate how the classical strength of double negation elimination emerges from right multiplicity by examining the relationship between derivations in natural deduction and sequent calculus and applications of their respective implication and negation rules. This clarifies how multiplicity constitutes a counterpart to double negation elimination.

[1] A.G. Dragalin, ***Mathematical Intuitionism: Introduction to Proof Theory.*** Translations of Mathematical Monographs, American Mathematical Society, (1988).

[2] D.M. Gabbay and N. Olivetti, ***Goal Directed Proof Theory.*** Applied Logic Series, Springer Dordrecht, (2000).

[3] G. Gentzen, *Untersuchungen über das logische Schließen.*, ***The Collected Papers of Gerhard Gentzen*** (Szabo, M. E., editor), North-Holland Publishing Company, Amsterdam, (1969), pp. 68–131.

[4] S.C. Kleene, ***Introduction to Metamathematics.*** North-Holland Publishing Company, (1952).

[5] S. Maehara, *Eine Darstellung der Intuitionistischen Logik in der Klassischen.*, ***Nagoya Mathematical Journal***, vol. 7 (1954), pp. 45–64.

[6] P. Milne, *Harmony, purity, simplicity and a "seemingly magical fact."*, ***The Monist***, vol. 85 (2002), no. 4, pp. 498-534.

[7] V. de Paiva and L. Pereira, *A short note on intuitionistic propositional logic with multiple conclusions.*, ***Manuscrito***, vol. 28 (2005), no. 2, pp. 317–329.

[8] G. Restall, *Structural rules in natural deduction with alternatives.*, ***Bulletin of the Section of Logic***, vol. 52 (2023), no. 2, pp. 109–143.

▸ TOM DE JONG, NICOLAI KRAUS, AREF MOHAMMADZADEH*, AND FREDRIK NORDVALL FORSBERG, *Generalized decidability via Brouwer ordinals.*
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Decidability and semidecidability are at the heart of computer science. If we think of "decidable" as "the answer is discoverable in finitely many (i.e. less than ω , the smallest infinite ordinal) steps," then "semidecidable" means that the answer is discoverable in less than $\omega + 1$ steps. This formulation allows for a finer hierarchy than just "decidable, semidecidable, undecidable." For example, if every $P(i)$ is semidecidable for each $i \in \mathbb{N}$, then $\forall i.P(i)$ may not be semidecidable, but its answer is discoverable in ω^2 steps: each $P(i)$ requires at most ω steps, and there are ω instances. In the context of constructive mathematics, where P is decidable if $P \vee \neg P$ and semidecidable if there exists a binary sequence that is somewhere 1 iff P holds, we suggest and study a framework in which the above and other statements can be made precise and proved. We work in homotopy type theory, using Brouwer ordinals to specify the level of decidability of a property. In this framework, we express the property that a proposition is α-decidable, for an ordinal α, and show that it generalizes decidability and semidecidability. Further generalizing known results, we show that α-decidable propositions are closed under binary conjunctions and discuss for which α they are closed under binary disjunction. We prove the result about countable meets alluded to in the first paragraph, and results for countable joins and iterated quantifiers. All our results are formalized in cubical Agda.

▸ COSIMO PERINI BROGI*, STELLA SPADONI, AND ROCCO DE NICOLA, *On protocol security via computability logic.*
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Experience in cybersecurity shows that communication protocol design is exceptionally error-prone: security weaknesses often arise less from defects in cryptographic primitives than from flawed protocols, owing to the incorrect logical interplay among agents and potential adversaries in a network. Empirical evidence likewise suggests that 'verification is only as sound as the language we used for it' [1].

We propose computability logic (CoL) [2] as a formal foundation for modelling, analysing and verifying secure communication protocols by treating specifications as interactive games between computational agents.

We argue that CoL naturally captures protocol dynamics and formally identifies structural invariants that underlie classes of vulnerabilities and attacks. Moreover, its constructive character favours automated strategy extraction and synthesis of correct-by-construction executable artefacts from verification proofs while preserving the game-theoretic, interactive nature of protocols.

We consider a CL4-based case study we presented at ITASEC 2026 to show the feasibility of our approach on representative protocol patterns, and outline theoretical and practical directions for extending the method to broader protocol families. This work in progress aims to advance logical verification methods for real-world secure

communication.

[1] Ross J. Anderson, ***Security engineering – A guide to building dependable distributed systems (3rd ed.)***, Wiley, 2020.

[2] Giorgi Japaridze, *Introduction to computability logic*, ***Annals of Pure and Applied Logic***, vol. 123 (2003), no. 1–3, pp. 1–99.

▸ PABLO DONATO, *Towards a Curry-Howard correspondence for existential graphs.*
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In this talk, I will introduce "scroll nets", a novel diagrammatic formalism for representing proofs and programs. As the name indicates, it is based on the "scroll", a topological notation for logical implication invented by Charles S. Peirce at the end of the 19th century for his system of existential graphs (EGs) [3]. In order to obtain a proper notion of static proof object from the inference rules of EGs, we add on top of the scroll a graphical syntax inspired by proof nets [1], which also acts as a form of term annotation in the style of type theory.

I will first motivate the notation from a philosophical standpoint, arguing that it captures the smallest motions of deductive reasoning in a very natural way, abstaining from any symbolic means of representation. I will then show how to express proofs in minimal implicative logic by simulating the rules of natural deduction. Through the Curry-Howard correspondence, this will lead us to identify a notion of detour that generalizes that of the simply-typed λ-calculus, making scroll nets into an expressive computational framework. If time remains, I will illustrate how to capture: 1. classical logic and intuitionistic disjunction by considering a horizontal generalization of the scroll first proposed by Oostra [2]; 2. intuitionistic subtraction [4] through a further vertical generalization of my own.

[1] Jean-Yves Girard, *Linear logic* ***Theoretical Computer Science*** vol. 50 (1987), no. 1, pp. 1–101

[2] Arnold Oostra, *Los gráficos Alfa de Peirce aplicados a la lógica intuicionista*, ***Cuadernos de Sistemática Peirceana***, 2010, no. 2, pp. 25–60.

[3] Charles Sanders Peirce, *Prolegomena to an Apology for Pragmaticism*, ***The Monist***, vol. 16 (1906), no. 4, pp. 492–546.

[4] Cecylia Rauszer *Semi-Boolean Algebras and Their Applications to Intuitionistic Logic with Dual Operations* ***Fundamenta Mathematicae*** vol. 83 (1974), pp. 219–249

▸ NICOLETA DUMITRU AND LAURENŢIU LEUŞTEAN*, *Effective methods: logic and optimization.*
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This talk presents a recent application in optimization of proof mining, a research direction in applied proof theory developed by Ulrich Kohlenbach beginning with the 1990s.

Let X be a normed space, $T : X \to X$ a nonexpansive mapping with fixed points, $f : X \to X$ a ρ-contraction for some $\rho \in [0, 1)$, $(r_n)_{n\in\mathbb{N}}$ a sequence in X, and $(\alpha_n)_{n\in\mathbb{N}}$, $(\beta_n)_{n\in\mathbb{N}}$, $(\delta_n)_{n\in\mathbb{N}}$ sequences in $[0, 1]$ such that for all $n \in \mathbb{N}$, $\alpha_n + \beta_n + \delta_n \leq 1$.

We study the asymptotic behaviour of the iteration

$$x_0 \in X, \qquad x_{n+1} = \delta_n f(x_n) + \alpha_n x_n + \beta_n T x_n + r_n, \tag{1}$$

which we refer to as the *inexact generalized Halpern iteration.* If f is a constant mapping, then (x_n) reduces to an iteration studied recently by Kanzow and Shehu [2]. Moreover, if we set, for all $n \in \mathbb{N}$, $\alpha_n = 0$, $\beta_n = 1 - \delta_n$, and $r_n = 0$ in (1), we obtain a viscosity version of the Halpern iteration studied by Xu [7] in Banach spaces and later by Sabach and Shtern [6] in Hilbert spaces, who called it the sequential averaging method (SAM).

We obtain quantitative and qualitative results on asymptotic and T-asymptotic regularity of the inexact generalized Halpern iteration by extending to this iteration the proof mining methods developed in [5, 3, 4] for the Halpern iteration. Furthermore, we compute rates of (T-)asymptotic regularity for particular choices of the parameter sequences, and for one of them, we obtain linear rates as an application of a lemma due to Sabach and Shtern [6].

[1] N. Dumitru and L. Leuştean, *Quantitative asymptotic regularity and T-asymptotic regularity for the inexact generalized Halpern iteration*, arXiv:2603.17105, 2026.

[2] C. Kanzow and Y. Shehu, *Generalized Krasnoselskii–Mann-type iterations for nonexpansive mappings in Hilbert spaces*, ***Computational Optimization and Applications***, vol. 67 (2017), pp. 595–620.

[3] U. Kohlenbach, *On quantitative versions of theorems due to F.E. Browder and R. Wittmann* ***Advances in Mathematics***, vol. 226 (2011), pp. 2764–2795.

[4] U. Kohlenbach and L. Leuştean, *Effective metastability of Halpern iterates in CAT(0) spaces* ***Advances in Mathematics***, vol. 231 (2012), pp. 2526–2556.

[5] L. Leuştean, *Rates of asymptotic regularity for Halpern iterations of nonexpansive mappings*, ***Journal of Universal Computer Science***, vol. 13 (2007), pp. 1680–1691.

[6] S. Sabach and S. Shtern, *A first order method for solving convex bilevel optimization problems*, ***SIAM Journal on Optimization***, vol. 27 (2017), pp. 640–660.

[7] H.-K. Xu, *Viscosity approximation methods for nonexpansive mappings*, ***Journal of Mathematical Analysis and Applications***, vol. 298 (2004), pp. 279–291

► MATTHIAS EBERL, *Formalizing indefinite extensibility.*
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We present a formalization of indefinite extensibility (i.e.) based on a dynamic, semantic approach. The underlying framework is type theory, and we develop the account in three stages of increasing strength: first-order logic (FOL), simple type theory (STT), and finally dependent type theory (DTT). Our notion subsumes Dummett's understanding of i.e., in the sense that a reference to the totality of all objects of some type — that is, a universal quantification — may increase the extension of this type. If the underlying notion of extensional definiteness is that only finite totalities are definite, this yields a formalization of the potential infinite as an indefinitely extensible finite, reflecting a finitistic viewpoint.

For FOL this approach has been fully developed in [1], with origins in [4]. The essential idea is that the domains of quantification may increase within a single formula. Since sets are by definition extensionally definite, higher-order concepts cannot be captured in ZFC set theory but can be treated in higher-order logic, and more generally, in type theory. Most of the model-theoretic notions for the higher-order case have been developed for STT in [2] and [3]. The core idea is to generalize the notions of direct and inverse systems and their limits. For instance, from a potentialist's perspective the natural numbers form a direct system $(\mathbb{N}_i)_{i>0}$ with $\mathbb{N}_i = \{0, \ldots, i-1\}$ and stages $1, 2, \ldots$. The key shift in perspective is that the limit construction does not terminate the extension process but yields only a context-dependent intermediate stage. Limits are understood as "sufficiently large" stages.

First components of the approach have been developed for DTT. The main aim here is to obtain a model that avoids a hierarchy of conceptually identical universes and instead uses a single increasing universe. Such a model then allows an interpretation of *type* : *type*. The model naturally has a restriction on syntactic expressiveness, so that not every typable term is interpretable, thereby avoiding inconsistencies. This, in turn, permits extensions of proof assistants such as Coq, Lean, or Agda in which certain types can be declared as indefinitely extensible, and from which one can then extract stages (bounds) from terms.

[1] Matthias Eberl, *A Model Theory for the Potential Infinite*, ***Reports on Mathematical Logic***, vol. 57 (2022), pp. 3–30.

[2] ——— *Higher-order concepts for the potential infinite*, ***Theoretical Computer Science***, vol. 945 (2023), Article 113667.

[3] ——— *A Reflection Principle for Potential Infinite Models of Type Theory*, ***29th International Conference on Types for Proofs and Programs (TYPES 2023)*** (Kesner, Delia and Reyes, Eduardo Hermo and van den Berg, Benno), Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2024, pp. 6:1–6:20.

[4] Jan Mycielski, *Locally Finite Theories*, ***Journal of Symbolic Logic***, vol. 51 (1986), no. 1, pp. 59–62.

▸ MIRKO ENGLER, *Semantic determinacy and self-interpretations of theories.*
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Drawing on ideas originating with [1], I motivate the philosophical standpoint that a syntactic interpretation of a theory T in a theory S determines the meaning of T relative to the meaning of S. In particular, an interpretation of T in itself determines its own meaning. This motivates the view that a theory may be regarded as *semantically determined* if there exists—up to a suitable notion of equivalence—exactly one self-interpretation. I then investigate the logical landscape generated by the various resulting notions of determinacy.

If a theory T admits, up to a - in T definable homotypy (cf. [3, ch. 5.4]) - only one self-interpretation, then T has *a-rigid* self-interpretations. If the relevant homotypy is (parametrically) definable only within each model of T, then T has *b-rigid* self-interpretations. In addition to regular interpretations, I also consider faithful interpretations, as well as retractions, in the sense studied by [4].

I present a series of results relating this proposed terminology to established notions of semantic determinacy, such as categoricity and κ-categoricity, as well as to more recently introduced concepts, including those proposed by [2]. I also examine theories that are intuitively regarded as semantically determined. Among other results, I show that theories like Successor Arithemtic, RCF and ACF_0 have a-rigid self-interpretations, whereas PA and ZF have a-rigid self-retractions. Based on the observation that PA has an extension with a-rigid self-interpretations (True Arithmetic), I discuss whether this might not be expected for ZF as well, and where difficulties might arise.

[1] Rudolf Carnap, ***Logical Syntax of Language***, Kegan Paul, 1937.

[2] Ali Enayat and Mateusz Lelyk, *Categoricity-like properties in the first order realm*, ***Journal for the Philosophy of Mathematics***, vol. 1 (2024), pp. 63–98.

[3] Wilfried Hodges, ***Model Theory***, Cambridge University Press, 1993.

[4] Albert Visser, *Categories of theories and interpretations*, ***Logic in Tehran. Lecture Notes in Logic***, (Ali Enayat, Iraj Kalantari and Mojtaba Moniri, editors), vol. 26, Cambridge University Press, 2006, pp. 284–341.

▶ ÁLVARO DÍAZ RAMOS*, GARRETT ERVIN, AND SAHARON SHELAH, *Generalized sums of linear orders.*
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Generalizing the arithmetic operations on the ordinals and cardinals, in the early 20th century, Hausdorff, followed by Whitehead and Russell, put forth the basic operations of addition and multiplication of linear orders. These were later studied in more general settings by Tarski and Birkhoff. What's surprising about this early period is that none of these authors ever attempted to justify their focus on these operations, beyond invoking the previous success of their study on ordinals. In fact, there has never been a formal justification given for this choice of operations.

In this talk, I will discuss our attempt to justify the naturality of the sum of linear orders by appealing to the categorical notion of sum provided by the coproduct. As it turns out, the category of linear orders in general lacks coproducts, so we develop a general theory of sums of linear orders, which are natural extensions of the coproduct in this setting. I will isolate algebraic and order-theoretic properties satisfied by the traditional sum of linear orders, and identify different sums exhibiting them. The most prominent and well-behaved of these are the simple sums, which emerge from sum-generating classes: classes of orders with a very rich algebraic structure. Most of the properties of the simple sums will in fact not hold in general, as I will show by introducing the method of complicated classes, a general approach to constructing unstructured sums.

Finally, I will discuss commutative sums of well-orders, which offer new proofs to many of the ordinals' order-theoretic properties — in particular, giving strict bounds on the possible sums of two ordinals. I will also discuss a recent extension of sums to other categories, including those of groups and modules.

▶ FÈLIX FRIGOLA GONZÁLEZ*, JOOST J. JOOSTEN, VICENT NAVARRO ARROYO, AND COSIMO PERINI BROGI, *On the Goldblatt-Thomason theorem for interpretability logic.*
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A theory U interprets a theory V in the sense of Tarski-Mostowski Robinson [5] roughly when there is a structure preserving translation j so that under this translation any theorem of V becomes a theorem of U, in symbols: $\exists j\ \forall\varphi(\Box_V\varphi \to \Box_U\varphi^j)$. Very similar to provability logic we can thus define the propositional interpretability logic of a theory $\mathbf{IL}(T)$. Even though interpretability is a Σ^0_3 complete notion [4], IL(T) is often a PSPACE decidable logic ([2]). Since its introduction in the 1980's, the modal theory of interpretability logics and related has become a mature field with so-called Veltman frames and models being the predominant relational semantics. Contrary to the case of provability logic, different sound theories can have different corresponding interpretability logics. These different logics are typically defined by adding axiom schemes over a base logic IL and over IL-frames these schemes give rise to so-called frame conditions and correspondences. Not all frame conditions are however modally definable. Ad-hoc methods aside [1], a general way of characterising modal definability is given by a so-called Goldblatt-Thomason Theorem. In this paper we comment on the proof of such a new theorem in the realm of interpretability logic with a special focus on the new techniques required.

[1] F. Frigola Gonzalez, J. J. Joosten, V. Navarro Arroyo, and C. Perini Brogi, *Ultrafilter Extensions for Veltman Semantics*, arXiv:2603.16754 (2026).

[2] L. Mikec, *Complexity of the interpretability logics ILW and ILP*, ***Logic Journal of the IGPL***, vol. 31 (2023), no. 1, pp. 194–213.

[3] L. Mikec, F. Pakhomov, and M. Vukovic, *Complexity of the interpretability logic IL*, ***Logic Journal of the IGPL***, vol. 27 (2019), no. 1, pp. 1–7.

[4] V. Y. Shavrukov, *Interpreting Reflexive Theories in Finitely Many Axioms*, ***Fundamenta Mathematicae***, vol. 152 (1997), pp. 99–116.

[5] A. Tarski, A. Mostowski, and R. Robinson, ***Undecidable Theories***, North-Holland, 1953.

▶ MASATO FUJITA, *Connected components of sets definable in d-minimal expansion of the real field*.
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Let $\mathfrak{R}$ be a d-minimal expansion of the ordered real field, that is, for every structure $\mathcal{M} = (M, <, +, \cdot, \dots)$ elementarily equivalent to $\mathfrak{R}$, every definable subset of M either has a nonempty interior or is a union of finitely many discrete sets [2]. A connected component of a subset of $\mathbb{R}^n$ definable in $\mathfrak{R}$ is not necessarily definable. However, $\mathfrak{R}$ can be expanded to a d-minimal structure $\mathfrak{R}^\natural$ so that every connected component of sets definable in $\mathfrak{R}$ is definable in $\mathfrak{R}^\natural$. Roughly speaking, a multi-cell proposed in [1] is a definable set which is union of countably many cells. A key fact used for the construction of $\mathfrak{R}^\natural$ is that sets definable in $\mathfrak{R}$ is partitioned into finitely many multi-cells.

[1] M. Fujita, *Almost o-minimal structures and $\mathfrak{X}$-structures*, ***Annals of Pure and Applied Logic***, vol. 173 (2022), 103144.

[2] C. Miller, *Tameness in expansions of the real field*, ***Logic Colloquium '01*** (M. Baaz, S. -D. Friedman and J. Krajíček, editors), Cambridge University Press, 2005, pp. 281–316.

▶ GIORGIO GENOVESI, *The tree interpretation and transfinite recursion.*
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The interpretation of **ZFC** without powerset in the theory of second order arithmetic can be divided into two steps. The first is a tree interpretation, sets are interpreted as well founded trees up to an appropriate equivalence relation needed to ensure extensionality. This gives a theory which satisfies Axiom Beta, that is, the statement that every well founded relation has a collapsing function. The second interpretation is the restriction to an initial segment of the constructible universe. These interpretations give a nice correspondence between subsystems of arithmetic and systems of set theory without the powerset axiom.

We go over some generalizations to this interpretation to systems of higher order arithmetic with restricted comprehension and theories of sets with bounded iterations of the powerset of ω. We also show that Axiom Beta implies the scheme of Elementary Transfinite Recursion as well as the totality of the primitive recursive set function. These results are part of a joint work with Emanuele Frittaion.

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Gödel's second incompleteness theorem is usually understood as exhibiting a gap between provability and truth [4]. We present a mathematically precise alternative reading in which the gap lies between two notions of consequence internal to a single arithmetical theory.

Working within the proof-theoretic semantics of Sandqvist [6], in which a semantic consequence relation $\Vdash$ is defined compositionally via support in bases of atomic inference rules, we show that this relation and derivability ($\vdash$) diverge for any sufficiently strong, recursively axiomatisable arithmetic theory A in the standard signature $\sigma = \langle 0, S, +, \times \rangle$: one has $A \Vdash \mathrm{Con}(A)$, even though $A \nvdash \mathrm{Con}(A)$. More generally, we establish the uniform reflection principle $A \Vdash \mathrm{Prov}_A(\ulcorner \varphi \urcorner) \to \varphi$ [2].

The proof proceeds by induction on codes of A-proofs, analysing the compositional clauses of the support relation. A key new observation is that the finiteness of the signature of arithmetic is what precisely controls the relationship between support and derivability: it prevents the completeness direction of the soundness and completeness theorem [3] from going through, so that semantic support is strictly stronger than derivability. Gödel's theorem is not contradicted; rather, incompleteness arises exactly at the point where completeness breaks down.

We further show that the framework admits an elementary consistency proof for PA [1]: ordinary induction over the natural numbers suffices to construct a consistent base supporting the axioms of PA, yielding $\mathsf{PA} \nVdash \bot$ by soundness. This stands in notable contrast to Gentzen's use of transfinite induction up to ε_0.

Together, these results establish that Gödel's incompleteness theorems can be understood as a separation between derivability and proof-theoretic semantic consequence, rather than as a gap between syntactic provability and truth in an independently given model [5].

[1] Alexander V. Gheorghiu, *Classical arithmetic without bivalence*, arXiv:2506.22326, 2026.

[2] ——— *On the concept of arithmetic consequence*, arXiv:2603.09900, 2026.

[3] ——— *Proof-theoretic semantics for first-order logic*, ***Logic Journal of the IGPL***, vol. 33 (2025), no. 5.

[4] Kurt Gödel, *Some basic theorems on the foundations of mathematics and their implications*, ***Collected Works: Unpublished Essays and Lectures*** (Solomon Feferman, John W. Dawson Jr., Warren Goldfarb, Charles Parsons, and Robert N. Solovay, editors), vol. III, Oxford University Press, 1995, pp. 304–323.

[5] Panu Raatikainen, *Gödel's incompleteness theorems*, ***The Stanford Encyclopedia of Philosophy*** (Edward N. Zalta and Uri Nodelman, editors), Metaphysics Research Lab, Stanford University, Spring 2026 ed., 2026.

[6] Tor Sandqvist, *Classical logic without bivalence*, ***Analysis***, vol. 69 (2009), no. 2, pp. 211–218.

▸ ALESSANDRO GIGLIA, *Against the equivalence of potentialist and actualist set theories.*
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According to potentialists, the current axiomatizations of the iterative conception of set fail to capture the idea of indefinite extensibility. To address this issue, potentialists present new theories, expanded with modal and plural resources. Actualists by contrast, argue that the classical axiomatizations are already fine. The equivalence thesis poses that actualist and potentialist set theories do not disagree, but are rather equivalent ways to express the same facts. The equivalence thesis has recently gained its strongest support in [1] where a result of near-synonymy between a potentialist and an actualist level theory is presented to support it.

This paper aims to show that such an argument can be resisted. More precisely, we argue that near-synonymy is not strong enough to achieve theoretical equivalence, and, even more importantly, we prove that the more substantive and widespread result of full-synonymy does not hold between any potentialist and actualist theory

We argue that near synonymy is not strong enough to achieve theoretical equivalence for three reasons: it is not an equivalence relation, it says nothing about the intended interpretations of the two theories, and the interpretations involved do not preserve important features of the theories, such as the complexity of the formulas.

We hence prove the non-synonymy of potentialist and actualist set theories by comparing them within the same logical vocabulary. The trick is to embed actualist theories in the modal logic *triv*, the logic obtained by adding to FOL the axiom $\Box\phi \leftrightarrow \phi$. The resulting theories are indeed proven to be equivalent to their non-modal counterparts, for the modal logic *triv* is indeed equivalent to FOL itself. When doing so, we clearly see that the equivalence thesis does not hold for an obvious reason: actualist theories prove much more than the potentialist theories, for the logic of the potentialist theory is a fragment of the logic of the actualist theory.

[1] Tim Button, *Level Theory Part 2*, ***The Bulletin of Symbolic Logic***, vol. 27 (2021), no. 4, pp. 436–460.

▸ ARNOLD GRIGORIAN, *Justifying homotopical logic.*
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The talk will be divided into two parts. The first one will be dedicated to the notion of internal language in category theory and its foundational significance. The second part will use the model-theoretic viewpoint to justify the logical status of the Univalence Axiom. Finally, the rest of the talk will be dedicated to connecting both of the approaches together in order to justify the idea behind homotopical logic.

Certain categories with rich enough structure (like various kinds of topoi) can be used as a model of a "mathematical universe". One can differentiate the use of the internal language of the category in question, i.e. 'reasoning within the universe', with 'meta-theoretical' or external reasoning (which is more similar to the ordinary style of mathematical reasoning), that is,'reasoning about the universe.'

This opens a possibility for various conceptually interesting interactions. Proving statements internally allows for the generalization of obtained results to a certain extent. However, I will give a few examples where external and internal presentations of the same concept do not coincide and why this distinction is important for modern practice regarding foundations of mathematics, such as univalent foundations.

As one can use set-theoretic constructions to encode all mathematics inside the hierarchy of sets, the same can supposedly be done with certain categories (e.g., (inf,1)-topoi). More importantly, if one uses intuitionistic principles internally, but the meta-theoretical is classical, the proof as a whole can't be considered constructive. As noted by T. Coquand [3], initial work on the simplicial model of univalent foundations used classical meta-theory, which resulted in non-constructive proofs. Thus, internal reasoning can prevent making unwarranted statements. Furthermore, it allows us to use an appropriately defined internal language as a tool to internalize principles of meta-theoretical nature, such as homotopy-invariance, shaping the idea behind homotopical logic (borrowing the name from A. Joyal's talk [4]).

The Univalence Axiom states that $=_U (A, B) \cong (A \cong B)$. The axiom was first introduced by Voevodsky and was motivated by the idea of homotopy theory, i.e., everything is considered under "homotopy equivalence". Another way to read the axiom is captured in S. Awodey's slogan for mathematical structuralism that "isomorphic structures can be identified".

In the homotopical interpretation of Martin-Löf's type theory, types are interpreted as spaces, i.e., they are essentially homotopy types of spaces. The introduction of the Axiom of Univalence into type theory allows to give a formal status to the intuition that any object is invariant under the homotopy equivalence, i.e. it internalizes the aforementioned external meta-theoretical principle.

The general idea goes as follows. Given a first-order language, one can formulate a list of axioms in a given fragment of logic $\mathbb{T}$. $\mathbb{T}$, then, is modeled by some mathematical objects. For example, having ZFC as a theory, it is modeled by the von Neumann universe $\mathbf{V}$, which is constructed using the meta-theory. In particular, $\mathbf{V}$ is constructed using set theory as a semi-informal meta-language and, consequently, ZFC will be the object language. From the foundational point of view, models (or in this case, $\mathbf{V}$) live in some set-theoretic universe, which is the case in the set-theoretical model theory.

From the perspective of category-theoretic semantics, this just means that "traditional" set-theoretical models "live" in the category **Sets**, i.e., semantics is a functor from some category that represents syntax of a given theory to the universe **Sets**. Instead of **Sets**, we can consider something with more structure, obtaining other kinds of models.

The addition of homotopical interpretation to type theory is essentially about interpreting identity as a homotopy equivalence. If two types are homotopy equivalent, they are identical, i.e., there exists an identity type between them. Homotopy theory is a study of objects invariant under continuous deformation, which presupposes a weaker notion of identity.

Finally, one can see that it makes perfect sense that the universe corresponding to the homotopical meta-theory above is not **Sets** and set-theory. In particular, for the intensional dependent type theory with the Univalence Axiom, correct models can be categorically described as infinity topoi. HoTT, as a theory, is the presupposed internal language of these categories. The fact that the Univalence Axiom holds only in "homotopical" models, makes the shift from set-theoretical foundations to category theory foundationally justified.

On the one hand, the example above gives a definitive positive answer to the question why one needs internal language in order to distinguish between different meta-theoretical principles since the presupposed internal language of the univalent universes is not set-theoretic.

On the other hand, the categories for which HoTT are supposed to be the internal language are characterized by weaker invariance criteria. This poses an interesting question about the logical status of the Univalence Axiom and the justification for homotopical logic.

[1] Steve Awodey, *Structure in Mathematics and Logic: A Categorical perspective*, ***Philosophia Mathematica***, vol. 4 (1996), no. 3, 209–237.

[2] Ingo Blechschmidt, *Using the internal language of toposes in algebraic geometry*, arXiv.2111.03685, 2021.

[3] Thierry Coquand, Fabian Ruch, and Christian Sattler, *Constructive sheaf models of type theory*, arXiv:1912.10407, 2021.

[4] André Joyal. *Remarks on homotopical logic*, ***The Homotopy Interpretation of Constructive Type Theory*** (Oberwolfach Reports), (Steve Awodey, Richard Garner, Per Martin-Löf, and Vladimir Voevodsky, editors), 2011, pp. 627–630.

[5] Chris Kapulkin and Peter LeFanu Lumsdaine, *The simplicial model of Univalent Foundations (after Voevodsky)*, ***Journal of the European Mathematical Society***, vol. 23 (2021), no. 6, pp. 2071–2126.

[6] Gerhard Osius, *The internal and external aspect of logic and set theory in elementary topoi*, ***Cahiers de topologie et geometrie differentielle***, vol. 15 (1974), no. .2, pp. 157–180

[7] Andrei Rodin, *Axiomatic Method and Category Theory*, Synthese Library, vol. 364, Springer Nature, 2014.

▸ JAMES E. HANSON, *Small large cardinals and neostability theory.*
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Neostability theory is the branch of model theory that studies certain classes of combinatorially tame first-order theories and more generally local combinatorial tameness within arbitrary theories. These tameness notions are often intimately linked to the behavior of indiscernible sequences. We will discuss some recent applications of certain small large cardinals (between ineffable and Erdős) in model-theoretic neostability theory and in particular their use in building certain special indiscernible sequences.

▸ ALEKSI HONKASALO, *Modal alternative set theory.*
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Petr Vopěnka's Alternative Set Theory (AST) was proposed as an alternative account of infinity [2]. The axioms of AST include both sets and classes and it is a conservative extension of finite ZF [1]. Sets in AST are all finite in the Cantorian sense, but AST treats some of sets as "naturally infinite". These sets contain subclasses which are not sets. These subclasses are called proper semisets. Proper classes are infinite both in the Cantorian sense as well from the AST's viewpoint, but Vopěnka contends that they are merely potentially infinite.

However, the motivations behind AST has not been widely understood or accepted. This presentation is a work-in-progress report of an investigation into the question of if AST's treatment of infinity can be clarified by using modal logic. The leading idea of the modal treatment of the axioms of is to start with a universe sets and claim that this universe can be extended to a universe which contains proper classes. To make room for proper semisets extensions may also contain non-standard elements as sets.

The modal treatment of the axioms will also allow for comparison between the treatment of potential infinity in AST and potentialist set theories as well as hopefully gain more insight into the notion of natural infinity.

[1] Antonín Sochor, *Metamathematics of the alternative set theory III*, ***Commentationes Mathematicae Universitatis Carolinae***, vol. 24 (1983), no. 1, pp. 137–154.

[2] Petr Vopěnka, ***Mathematics in the Alternative Set Theory***, B.G. Teubner Verlagsegellschaft, 1979.

▸ DANIEL ISAACSON, *Dedekind's axiomatisation of arithmetic as a case of informal rigour.*
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Georg Kreisel propounded the idea(l) of "informal rigour" in his paper "Informal rigour and completeness proofs" (1967), which he later characterised as "a venerable ideal in the broad tradition of analysing precisely common notions or, as one sometime says, notion implicit in common reasoning" (Church's Thesis and the Ideal of Informal

Rigour", 1987, p. 499). Identifying and conceptualising this element of mathematical practice constitutes an important contribution to the philosophy of mathematics. I will argue that the correctness and fruitfulness of Richard Dedekind's axiomatisation of arithmetic in *Was sind und was sollen die Zahlen?* (1888) was established by informal rigour in Kreisel's sense. I will formulate this argument in terms of the schematisation of informally rigorous arguments, in Kriesel's sense, by Walter Dean and Hidenori Kurokawa in their paper "On the methodology of informal rigour: set theory, semantics, and intuitionism" (to appear in the *Journal of Philosophical Logic*), with common notions $\mathcal{C}_1$ = "a is a natural number", $\mathcal{C}_2$ = "natural number a succeeds natural number b", $\mathcal{C}_3$ = "a is the sum of b and c", $\mathcal{C}_4$ = "a is the product of b and c", $\mathcal{C}_5$ = "a is less than b", and accepted (so in that sense precise) notions $\mathcal{P}_1$ = "a is a set (system)", $\mathcal{P}_2$ = "b is an element of a set a", $\mathcal{P}_3$ = "φ is a mapping from a set a to a set b", $\mathcal{P}_4$ = "a is the intersection of all the sets in b", and defined notion: "K is a chain with respect to set a and mapping φ". This schematisation provides a basis for establishing with informal rigour that Dedekind's axiomatisation of the natural number and his proof of the recursion theorem, and thereby the categoricity of his axiomatisation, are correct.

▸ GALINA KALEEVA, *Universal equivalence of linear groups over rings.*
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Two algebraic structures are said to be universally equivalent if they have the same universal theory (the set of all universal sentences). A classical question about linear groups is whether two groups $\mathrm{GL}_n(R_1)$ and $\mathrm{GL}_m(R_2)$ are equivalent in some sense (e.g., isomorphic or elementarily equivalent) if and only if $n = m$ and the rings R_1 and R_2 are equivalent in the same sense. In this talk, we discuss this question for universal equivalence of general linear groups over (possibly noncommutative) local, associative, unital rings with 1/2. It turns out that if $m, n \geqslant 3$, then the groups are universally equivalent if and only if $n = m$ and either the rings R_1 and R_2 are universally equivalent or the rings R_1 and R_2^{op} are universally equivalent.

[1] Galina A. Kaleeva, *Universal equivalence of general linear groups over local rings with* 1/2, ***Sbornik: Mathematics***, vol. 216 (2025), no. 10, pp. 1363–1374.

▸ BENJAMIN KOCH*, ELVIRA MAYORDOMO, ARNO PAULY, CÉCILIA PRADIC, AND MANLIO VALENTI, *On the computational strength of Hausdorff oracles.*
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Over the last decade, there have been several developments in geometric measure theory which were achieved via algorithmic information theory. The catalyst for this was the *point-to-set principle* of J. Lutz and N. Lutz [1], which established a correspondence between the Hausdorff and packing dimensions of subsets of $\mathbb{R}^n$ and the "effective" dimensions of their points. The effective dimension of a point is understood as the compressibility by oracles $A \subseteq \mathbb{N}$ of finite approximations of it. The point-to-set principle establishes the existence of oracles that describe the Hausdorff (or packing) dimension exactly; we call these Hausdorff or packing oracles respectively.

In this work, we investigate the computational strength of Hausdorff oracles in the sense of Weihrauch reducibility. We establish an equivalence between finding a Hausdorff oracle of a set in $\mathbb{R}^n$ and finding the sequence of covers which witnesses its classical Hausdorff dimension. We then show that Hausdorff oracles for $\mathbf{\Sigma}^0_1$, $\mathbf{\Pi}^0_1$, and $\mathbf{\Sigma}^0_2$ sets are computable from a name for the set in question. Furthermore, we establish a cone-avoidance result for Hausdorff oracles of countable sets. With regard to $\mathbf{\Pi}^0_2$ sets, we prove that establishing a lower bound (strict or otherwise) on the Hausdorff dimension of such a set is analytic-complete, while determining the dimension exactly is Weihrauch equivalent to $\mathbf{\Pi}^1_1$-CA. We also show that finding a Hausdorff oracle for a $\mathbf{\Pi}^0_2$ set is not Weihrauch reducible to $\mathsf{UC}_{\mathbb{N}^\mathbb{N}}$, unique choice on Baire space.

[1] Jack H. Lutz and Neil Lutz, *Algorithmic information, plane Kakeya sets, and conditional dimension*, ***ACM Transactions on Computation Theory***, vol. 10 (2018), no. 2, pp. 7:1–7:22

▸ ELIO LA ROSA, *Skolemization and Herbrand's Theorem for intermediate logics over predicate abstraction.*
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I present a new approach to Skolemization and proof of Herbrand's Theorem for intermediate logics based on predicate abstraction. Predicate abstraction provides terms with a scoping mechanism allowing for *de dicto* and *de re* reading of modalities. As Fitting [3] noticed predicate abstractions are useful conservative extensions also for modal logics which cannot represent such distinctions but also cannot represent complete Skolemizations otherwise. In this talk, I show how modalities can be absorbed into predicate abstractions to provide a conservative extension for Intuitionistic and other intermediate logics. Semantically, the interpretation of Skolem functions then corresponds to an intensional version of Bazz and Iemhoff's approach (e.g., [1]), using existence predicates. The term-scoping mechanism, however, allows for simplified proofs of completeness of Skolemization and a modular approach. The account is proof-theoretic, and based on the nested calculi for intermediate logics first presented by Ciabattoni et al. [2] and turned first-order by Lyon [4]. These calculi are thus extended to identity, predicate abstraction and non-rigid terms. Intuitionistic, Gödel-Dummett and Bunched Implication logics will be covered in their varying and constant domain versions.

[1] Matthias Baaz and Rosalie Iemhoff, *Eskolemization in Intuitionistic Logic*, ***Journal of Logic and Computation***, vol. 21 (2011), no. 4, pp. 625–638.

[2] Agata Ciabattoni, Lutz Strassburger and Matteo Tesi, *Taming Bounded Depth with Nested Sequents*, ***AIML 2022 - Advances in Modal Logic*** (Rennes, France), (David Fernández-Duque, Alessandra Palmigiano and Sophie Pinchinat, editors) College Publications, 2022, pp. 199–216.

[3] Melvin Fitting, *A Modal Logic Analog of Smullyan's Fundamental Theorem*, ***Mathematical Logic Quarterly***, vol. 19 (1973), no. 1, pp. 1–16.

[4] Tim Lyon, *Nested Sequents for Intermediate Logics: The Case of Gödel-Dummett Logics*, ***Journal of Logic and Computation***, vol. 33 (2023), no. 2, pp. 121–164.

▸ CHANWOO LEE AND S. KAAN TABAKCI*, *Model-theoretic inferentialism: the best of both worlds.*
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What makes logical and mathematical terms meaningful? What are the meanings

of logical and mathematical terms? We explore model-theoretic inferentialism, which combines an inferentialist answer to the metasemantic question (the former) and a denotation-based, model-theoretic answer to the semantic question (the latter). We characterize and defend model-theoretic inferentialism against rivaling accounts, especially orthodox denotationalism and orthodox inferentialism in logic and mathematics. Model-theoretic inferentialism has many variations, so we offer a way to classify different variations of model-theoretic inferentialism based on the normative basis of correct inferences and the ontological status of model-theoretic entities. Then, we introduce the Categoricity Problem: a major formal problem for model-theoretic inferentialism where the model-theoretic meanings of logical and mathematical terms are not uniquely determined by the inferences of the standard systems. We discuss the solutions in the literature to uncover how they match up with different approaches to model-theoretic inferentialism, since not every solution serves every form of model-theoretic inferentialism equally well. Based on these considerations, we develop three philosophical accounts of model-theoretic inferentialism, which will help us better comprehend the relevant results in philosophy of logic and mathematics.

[1] Nuel D. Belnap and Gerald J. Massey, *Semantic Holism*, ***Studia Logica***, vol. 49 (1990), no. 1, pp. 67–82.

[2] Dennis Bonnay and Dag Westerståhl, *Compositionality Solves Carnap's Problem*, ***Erkenntis***, vol. 81 (2016), no. 4, pp. 721–739.

[3] Tim Button and Sean Walsh, ***Philosophy and Model Theory***, Oxford University Press, 2018.

[4] Rudolph Carnap, ***Formalization of Logic***, Harvard University Press, 1943.

[5] James Garson, ***What Logics Mean***, Cambridge University Press, 2014.

[6] Julien Murzi and Brett Topey, *Categoricity by Convention*, ***Philosophical Studies***, vol. 178 (2021), no. 10, pp. 3391–3420.

[7] Greg Restall, *Truth Values and Proof Theory*, ***Studia Logica***, vol. 92 (2009), pp. 241–264.

[8] Sebastian Speitel, *Carnap's Categoricity Problem*, ***The Bulletin of Symbolic Logic***, to appear.

[9] S. Kaan Tabakci, *Categoricity Problem for LP and K3*, ***Studia Logica***, vol. 112 (2024), no. 6, pp. 373-1407,

[10] Jarred Warren, ***Shadows of Syntax***, Oxford University Press, 2020.

► PAUL BLAIN LEVY, *Broad infinity implies Mahlo assuming choice: a direct proof.*
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Mahlo's principle (aka Ord is Mahlo) says that the class of all regular limit ordinals is stationary [1, 3]. Although a well-known and useful axiom scheme, it is somewhat lacking in intrinsic plausibility. To remedy this situation, the recent paper [2] shows that (given ZFC) Mahlo's principle follows from an axiom scheme called Simple Broad Infinity that may be considered intrinsically plausible. (Of course such plausibility judgements are subjective.)

While the proof given in [2] for this inference was complicated and indirect, we now present an easy and direct proof.

We use the notation $\mathsf{Begin} \stackrel{\text{def}}{=} \{\}$ and $\mathsf{Make}(x, y) \stackrel{\text{def}}{=} \{\{x\}, \{x, y\}\}$. We write $\mathfrak{T}$ for the universal class, Set for the class of all sets (which may differ from $\mathfrak{T}$ if urelements are allowed), and Ord for the class of all ordinals (the least class such that every transitive subset is a member).

Simple Broad Infinity says that, for any function $F : \mathfrak{T} \to \mathsf{Set}$, there is a set X with the following properties:

- $\mathsf{Begin} \in X$.
- For any $x \in X$ and $y \in X^{Fx}$, we have $\mathsf{Make}(x, y) \in X$.

For a set U of ordinals, we write $\mathsf{ssup}U$ for its strict supremum. To show Mahlo's principle, it suffices to show that, for any function $H : \mathsf{Ord} \to \mathsf{Set}$, that there is an ordinal α with the following properties:

- $0 < \alpha$.
- For any $\beta < \alpha$ and $p : H\beta \to \alpha$, we have $\mathsf{ssup}(\{\beta\} \cup p_i \mid i \in H\beta) \leqslant \alpha$.

Sketch proof: construct a class of "pre-broad numbers", each of which has a rank, then convert $H : \mathsf{Ord} \to \mathsf{Set}$ to $F : \mathfrak{T} \to \mathsf{Set}$ by taking ranks of pre-broad numbers. The set of ranks of elements of the set generated is the desired ordinal, using AC to choose elements with each required rank.

[1] Azriel Lévy, *Axiom schemata of strong infinity in axiomatic set theory*, ***Pacific Journal of Mathematics***, vol. 10 (1960), no. 1, pp. 223–238.

[2] Paul Blain Levy, *Broad infinity and generation principles*, ***Notre Dame Journal of Formal Logic***, vol. 66 (2025), no. 1, pp. 79–141.

[3] Hao Wang, *Large sets*, ***Logic, foundations of mathematics, and computability theory***, Springer, 1977, pp. 309–333.

▸ DAVIDE MANCA, *Better-quasi-ordering sequences with the gap condition.*
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Being a well-quasi-order is a fundamental combinatorial property with important applications to logic and computer science. A classic result by Higman states that finite sequences with elements in a well-quasi-order form a well-quasi-order with respect to the relation induced by Higman embeddings [2]. Gordeev proved [1] that, for sequences with elements in a well-order, the theorem holds even under an additional requirement: a symmetrical version of the gap condition originally formulated by Friedman for labelled trees (see [4]). Under that condition, the consistency strength of the theorem increases considerably.

We prove that Gordeev's relation of symmetrical gap-embeddability satisfies the stronger property of being a better-quasi-order, a natural strengthening of well-quasi-orders introduced by Nash-Williams. In particular, our proof relies on a restricted form of Nash-Williams' better-quasi-ordering theorem for trees [3]. We also obtain an upper bound for the strength of our theorem in reverse mathematics, in terms of systems of partial impredicativity [5]. These results were obtained in collaboration with Patrick Uftring (University of the Bundeswehr Munich).

[1] Lev Gordeev, *Generalizations of the one-dimensional version of the Kruskal-Friedman theorem*, ***The Journal of Symbolic Logic***, vol. 54 (1989), no. 1, pp. 100–121.

[2] Graham Higman, *Ordering by divisibility in abstract algebras*, ***Proceedings of the London Mathematical Society***, vol. s3-2 (1952), no. 1, pp. 326–336.

[3] C.St.J.A. Nash-Williams, *On well-quasi-ordering infinite trees*, ***Mathematical Proceedings of the Cambridge Philosophical Society***, vol. 61 (1965), no. 3, pp. 697–720.

[4] Stephen G. Simpson, *Nonprovability of Certain Combinatorial Properties of*

Finite Trees, in: ***Harvey Friedman's Research on the Foundations of Mathematics*** (L.A. Harrington and M.D. Morley and A. Sĉêdrov and S.G. Simpson, editors), Vol. 117. Studies in Logic and the Foundations of Mathematics, Elsevier, North-Holland, Amsterdam, 1985, pp. 87–117.

[5] Henry Towsner, *Partial impredicativity in reverse mathematics*, ***The Journal of Symbolic Logic***, vol. 78 (2013), no. 2, pp. 459–488.

▸ SAMUELE MASCHIO, *A categorical account of relativization.*
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Relativization is one of the first techniques one learns to produce relative consistency models in set theory (see e.g. [2]). Once a formula $\varphi(x)$ is chosen, each formula of the language is translated by restricting quantifiers to the definable class $\{x|\,\varphi(x)\}$.

The aim of this talk is to generalize relativization to the framework of doctrines. In the field of categorical logic, doctrines are particular contravariant functors providing an algebraic account of a theory (see e.g. [3, 4]).

Here we propose two categorical approaches to relativization and we show some of their manifestations. In particular, we will recover, as an example, the relation between uniform realizability and Kleene realizability, as it is presented in [1], showing that such categorical counterparts of relativization allow to cover under the same construction a wide spectrum of relevant phenomena, beyond the strictly set-theoretical ones.

[1] Ulrich Berger and Paulo Oliva, *Uniform Realizability Interpretations*, ***Logics and Type Theory: Essays dedicated to Stefano Berardi on the occasion of his 1000000th birthday, EPTCS 441*** (Thorsten Altenkirch, Franco Barbanera, Ferruccio Damiani and Ugo de'Liguoro, editors), Electronic Proceedings in Theoretical Computer Science, 2026, pp. 46–61.

[2] Kenneth Kunen, ***Set Theory***, Studies in Logic and the Foundations of Mathematics 102, North-Holland, 1980.

[3] F. William Lawvere, *Adjointness in foundations*, ***Dialectica***, vol. 23 (1969), no. 3–4, pp. 281–296.

[4] Maria Emilia Maietti and Giuseppe Rosolini, *Quotient completion for the foundation of constructive mathematics*, ***Logica Universalis***, vol. 7 (2013), no. 3, pp. 371–402.

▸ RODRIGO MENA GONZÁLEZ, *Two logics for normative announcements.*
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Some philosophers have considered commands and authorisations as speech acts that increase or reduce the set of states of affairs (sometimes actions), that their addressees can obtain or perform in a given context (see, for example, [3], [4], [2], [1]). In this work, we present two logics aiming to capture that intuition, based on the same techniques developed to represent public announcements in Dynamic Epistemic Logic. Public announcements are taken as reducing the set of possible worlds accessible for agents by eliminating those which are incompatible with what is actually the case, as in [6]. Here, we take commands and authorisations as reducing or extending the permissibility sphere of agents, that is, the set of ideal worlds open to them. Although most similar systems take S5 as a base logic and perform model updates by changing a preference order among worlds (as in [7], [8]) or via model product operations (see, for instance, [5]), we instead use a weaker deontic logic in the vecinity of S4 and take a much direct approach. This faces some problems, since the elimination (and, in our case, also the addition of available possible worlds) requires the frame conditions on our models to be preserved by the update operations. A sound and complete axiomatic system close to that of [9] is introduced.

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▸ YIPING MIAO, *Generic reals and gauge dimensions.*
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Measure and category are two orthogonal notions. A typical element in a comeager set is generic, and a typical element in a conull set is random. There exists comeager null set, and the set of (Cohen) generic reals provide an example. The set also has Hausdorff dimension zero. However, the set has a perfect subset, suggesting largeness under certain measures. We consider an extended notion of Hausdorff dimension, using a gauge function to count how much an open ball contributes.

For any set of reals, the gauge profile of it is the set of gauge functions making the set measure positive. We will show that the complexity of the dense sets the generic meets corresponds nicely with the complexity of functions generating its gauge profile. Then we look at Sacks generics and Mathias generics. These two examples provide a comparison between gauge profile and the complexity of the collection of dense sets.

Time allows, we will also discuss the gauge profile of the set of reals compressible by 1/2. This is the random real analogue of the (Cohen) generic case. The set is known to have dimension 1/2 and our work shows the gauge profile is supported on its dimension.

▸ RUSSELL MILLER, *Computable elements within tree-presented structures.*
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Continuum-sized structures are often presentable as the set of paths through a computable subtree of $\omega^{<\omega}$, with the functions in the signature computed by Turing functionals on those paths. Such structures include the rings $\mathbb{Z}_p$ of p-adic integers, related groups such as $\widehat{\mathbb{Z}}^+$ and $\widehat{\mathbb{Z}}^\times$, the group $\mathrm{Sym}(\omega)$ of all permutations of ω, and all automorphism groups of computable structures. If quotients by Π^0_1 equivalence relations are allowed, then the fields $\mathbb{R}$ and $\mathbb{C}$ are also included. These *tree-computable presentations* are formally defined in [1].

Within such a structure, the computable paths form a countable substructure. In the case of the field $\mathbb{R}$, Korovina and Kudinov showed in [2] that the Turing degree spectrum of this substructure consists of precisely the high degrees (i.e., those $\boldsymbol{d}$ with $\boldsymbol{d'} \geq \mathbf{0''}$). In contrast, Morozov proved in [3] that the computable permutations of ω form a group whose degree spectrum is the upper cone above $\mathbf{0''}$. We offer some further results here, showing, for instance, that the spectrum of the group $\mathrm{Gal}_{\mathbf{0}}(\mathbb{Q})$ of computable automorphisms of the algebraically closed field $\overline{\mathbb{Q}}$ is contained in the upper cone above $\mathbf{0'}$.

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▸ JOACHIM MUELLER-THEYS, *On the possibility of logical constructivism.*
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I. Let L be any language with alternations $\phi \vee \psi$. Further, $\Phi \vdash \phi$ be any provability relation on L such that premises are provable, viz. $\phi \in \Phi \Longrightarrow \Phi \vdash \phi$.

Given the rules of $\vee$-introduction, $\Phi \vdash \phi$ v $\Phi \vdash \psi$ is a sufficient condition for $\Phi \vdash \phi \vee \psi$. In the discussion to his talk at the *Logica Universalis Webinar* (April 2022), A. Drago claimed that intuitionistic theories would have excluded middle only if they are complete. We could prove this indeed provided that $\Phi \vdash \phi$ v $\Phi \vdash \psi$ is a *necessary* condition for $\Phi \vdash \phi \vee \psi$ (too), as claimed in intuitionistic meta-logic.

However, reasonable such $\vdash$, we call them $\vee$-*constructive* or *alternativeist*, do not exist, as we have proven by means of universal logic and semantics.

II. We therefore evidently assume only that $\vdash$ *has some sound alternative semantics showing binary contingency*, videlicet there exists some satisfaction relation $\models \subseteq \Im \times L$ such that $I \models \phi \vee \psi \Longleftrightarrow I \models \phi$ v $I \models \psi$, $\Phi \vdash \phi \Longrightarrow \Phi \models \phi$ (where $\models = \models' \subseteq \wp(L) \times L$ is the Tarskian consequence relation derived from $\models$), and there are *simultaneously contingent* ϕ_0, $\psi_0 \in L$, viz. $I_1 \models \phi_0, \psi_0$; $I_2 \models \phi_0$, $I_2 \not\models \psi_0$; $I_3 \not\models \phi_0$, $I_3 \models \psi_0$; and $I_4 \not\models \phi_0$, ψ_0 for suitable $I_1, I_2, I_3, I_4 \in \Im$.

Let ϕ, ψ be simultaneously contingent. Then, particularly, ϕ, ψ are (consequentially) *independent*, viz. $\phi \not\models \psi$, $\psi \not\models \phi$. We proved that neither $\phi \vee \psi \models \phi$ nor $\phi \vee \psi \models \psi$ thence, which is why we call $\phi \vee \psi$ *essential* in this case. Thereby, since $\vdash$ is sound, $\phi \vee \psi \nvdash \phi, \psi$. By the rule for premises, $\phi \vee \psi \vdash \phi \vee \psi$.

Thus, for $\Phi_0 := \{\phi_0 \vee \psi_0\}$, $\Phi_0 \vdash \phi_0 \vee \psi_0 \not\Longrightarrow \Phi_0 \vdash \phi_0$ v $\Phi_0 \vdash \psi_0$, i. e. $\vdash$ is *not* generally $\vee$-constructive. Since $\vee$-constructivity is a feature of constructivity, $\vdash$ is not constructive.

III. We now assume that L is additionally equipped with negation $\neg : L \to L$ and that $\vdash \subseteq \wp(L) \times L$ *has some sound classical semantics showing contingency*, videlicet $\vdash$ has some sound alternative semantics such that $I \models \neg\phi \Longleftrightarrow I \not\models \phi$ and there is ϕ_0 with $I_1 \models \phi_0$, but $I_2 \not\models \phi_0$ for some I_1, I_2. Then $\phi_0 \vee \psi_0$ with $\psi_0 := \neg\,\phi_0$is essential, whence, as in II, $\vdash$ is not ($\vee$-)constructive.

IV. We may conclude that reasonable provabilities with constructive alternation do not exist. In particular, minimal and intuitionistic $\vdash$ (like IPC) are not alternativeist, since they are sound with respect to classical logical semantics.

A fully elaborated script is available.

▸ NAZERKE MUSSINA, INDIRA TUNGUSHBAYEVA, AND AIBAT YESHKEYEV, *The hybrids of holographic Jonsson theories and their Jonsson spectrum.*
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Let L be a first-order countable language of signature σ, σ be a finite predicate signature, T be a Jonsson L-theory [1], and C_T be a semantic model [3] of the theory T.

Let us define the hybrid of Jonsson theories [4], the holographic Jonsson theory and model of holographic Jonsson theory.

DEFINITION. Let T_1, T_2 be Jonsson L-theories, C_1, C_2 be their semantic models, respectively. A hybrid $H(T_1, T_2)$ of the first type is a theory $Th_{\forall\exists}(C_1 \times C_2)$ if it is Jonsson.

DEFINITION ([2]). A Jonsson theory T is called holographic if $S_n^J(T)$ is finite, where $n = \parallel \sigma \parallel$ and $S_n^J(T)$ is the set of all complete $\forall\exists$-types in n free variables.

DEFINITION. A model A of a holographic Jonsson theory T is called Jonsson-holographic if the following conditions hold: 1) $Th_{\forall\exists}(A)$ is a Jonsson theory; and 2) $Th_{\forall\exists}(A)$ is holographic.

THEOREM. *Let T_1, T_2 be holographic Jonsson theories. Suppose that there exists a hybrid of Jonsson theories $H(T_1, T_2)$, $JSp(C_{H(T_1,T_2)})$ is a Jonsson spectrum of $C_{H(T_1,T_2)}$. Then the following statements hold:*

1) There exists a theory $\Delta \in JSp(C_{H(T_1,T_2)})$ such that Δ is a holographic Jonsson theory;

2) If $\Delta_1, \Delta_2 \in JSp(C_{H(T_1,T_2)})$ and Δ_1, Δ_2 are holographic Jonsson theories such that the hybrid $H(\Delta_1, \Delta_2)$ exists, then this hybrid is also a holographic Jonsson theory and $H(\Delta_1, \Delta_2) \in JSp(C_{H(T_1,T_2)})$.

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▸ MORENIKEJI NERI, *Extracting bounds from proofs involving ultraproducts.*
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An important application of ultraproducts in mathematics is the derivation of uniform bounds for existential statements via their validity in ultrapowers. More precisely, in the setting of first-order logic, one has the following principle.

Theorem. *Let T be a first order theory and $\{\varphi_n\}$ a collection of first order formulas. Suppose that for all non-principal ultrafilters $\mathcal{U}$ and all first order structures $\mathcal{M}$ with $\mathcal{M}_{\mathcal{U}} \models T$, we have there exists $n \in \mathbb{N}$ such that $\mathcal{M}_{\mathcal{U}} \models \varphi_n$. Then there exists $N \in \mathbb{N}$ such that for all $\mathcal{M} \models T$ there exists $n \leq N$ with $\mathcal{M} \models \varphi_n$.*

The proof of this result is inherently *nonconstructive*, which raises the natural question of whether bounds obtained in this way can be made explicit. This question lies close to the heart of *proof mining* [4], a program in mathematical logic, initiated by Kohlenbach and collaborators, that seeks to extract effective quantitative information from proofs in mainstream mathematics.

Proof mining is underpinned by general proof-theoretic tools known as *logical metatheorems*, which guarantee that proofs carried out in suitable formal systems admit highly uniform computational content. These metatheorems not only provide algorithms for extracting this content, but also bounds their complexity in terms of the logical principles used.

In case studies, most notably the work of Simmons and Towsner [3], it has been observed that analyzing proofs based on Theorem 1 through the lens of existing metatheorems of proof mining often yields explicit uniform bounds. Building on earlier work of Günzel and Kohlenbach [2], we present a new metatheorem that explains this phenomenon in a systematic way. As an application, we derive new explicit bounds for results in [1] which we obtained through this methodology.

This is joint work with Ulrich Kohlenbach and Jin Wei.

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▸ EIKE NEUMANN, *On the complexity of quadratic iteration.*
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It is unknown whether there exists an algorithm to decide whether the complex number 0 escapes the closed disc of radius 2 under the iteration of the polynomial $f_c(z) = z^2 + c$ for a given $c \in \mathbb{Q}[i]$. A closely related problem is the long-standing open question whether the Mandelbrot set is computable [1]. This motivates more general questions of the form: can we decide whether a point escapes a set under the iteration of a quadratic map? Here, we answer one such question negatively.

Theorem. *For $d \geq 1$, let $\mathbb{H}_d = \left\{(x_1, \ldots, x_d) \in \mathbb{R}^d \mid x_d \geq 0\right\}$. There exist a positive integer d and a quadratic map $f\colon \mathbb{R}^d \to \mathbb{R}^d$, whose components are quadratic polynomials $f_1, \ldots, f_d \in \mathbb{Q}[x_1, \ldots, x_d]$, such that it is impossible to decide for a given point $x_0 \in \mathbb{Q}^d$ whether there exists $n \in \mathbb{N}$ with $f^{\circ n}(x_0) \in \mathbb{H}_d$.*

This theorem is proved by reduction from Hilbert's Tenth Problem via Laczkovic's sharpening [2] of Richardson's theorem [3]. Laczkovic's result yields a continuous-time analogue of this theorem, involving polynomial differential equations. We obtain the discrete-time version through a bespoke numerical integration scheme. In future work, it would be very interesting to study the same problem where the half-space $\mathbb{H}_d$ is replaced either by the hyperplane $\{(x_1, \ldots, x_d) \in \mathbb{R}^d \mid x_d = 0\}$ or the unit ball $\{(x_1, \ldots, x_d) \in \mathbb{R}^d \mid x_1^2 + \cdots + x_d^2 \leq 1\}$

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▸ RAGNA OEYNHAUSEN, *A truthmaker semantics for positive free logic.*
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This paper attempts to develop an exact truthmaker semantics for positive free logic. On this account, a truthmaker for a proposition is a *state* (a possible fact) that makes the proposition true. While, classically, an atomic statement is true iff the object denoted by the individual term falls under the given predicate, the current proposal analyses truth with reference to verifying facts. Consequently, the truthmaker approach is particularly suited to handle statements containing *empty* singular terms, i.e. singular terms that do not denote an object in the domain. Logics that admit empty-termed statements are called free; if empty-termed statements may be true, the logic is a positive free logic.

This paper adopts the exact truthmaker semantics developed in [1] and extends the proposal for first-order sentences, quantifiers, identity, and possibly empty singular terms. After providing the generalised semantical rules, a soundness and completeness result will be established. Along the way, it will be shown that the current proposal provides an exactification of the well-known dual-domain semantics for positive free logics (see [2] for a modern presentation). Translations between the two models will be provided. Finally, it is argued that the truthmaker account is preferable on philosophical grounds.

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▸ GIAN MARCO OSSO, *The Laver partition theorem for open sets: the nucleus of Ramsey property and determinacy?*
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I will present work in progress, joint with Alberto Marcone, on the computational and proof theoretic strength of a partition theorem related to Laver trees, which we called the Laver partition theorem. This theorem states that given any sufficiently definable set A, either A or its complement contains the body of some Laver tree. I will show that, for a given class Γ, the Laver partition theorem for Γ is a stripped down version of both the determinacy and the Ramsey property of sets in Γ. My talk will focus on the case where Γ is the class of open sets, the only case in which, in terms of reverse mathematics, the strength of determinacy coincides with that of the Ramsey property (this strength is precisely captured by the system ATR_0 of arithmetical transfinite recursion). I will present results towards answering the natural question: is the Laver partition theorem already at the level of ATR_0?

▸ LUDOVIC PATEY AND PAUL SHAFER*, *Bounded Ramsey's theorem for triples in computability theory.*
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Bounded Ramsey's theorem for 2*-colorings of* n*-tuples* restricts RT^n_2 to colorings where the homogeneous sets for color 1 are of bounded size. We investigate the computational content of $\mathsf{BRT}^3_{2,\ell}$, which states that every coloring $f\colon \mathbb{N} \to \{0,1\}$ with no homogeneous set for color 1 of size $\ell \geq 4$ has an infinite homogeneous set for color 0. Frittaion initiated the systematic study of bounded Ramsey's theorem. Soldà picked up the topic, proving in his thesis that $\forall\ell\,\mathsf{BRT}^3_{2,\ell}$ is cone avoiding and hence does not imply ACA_0. Recently, Le Houérou and Patey studied $\mathsf{BRT}^2_{2,\ell}$ for $\ell \geq 3$.

It is not difficult to see that $\mathsf{RT}^2_2 \leq_{\mathrm{c}} \mathsf{BRT}^3_{2,4}$ and that $\forall\ell\,\mathsf{RT}^2_\ell \leq_{\mathrm{c}} \forall\ell\,\mathsf{BRT}^3_{2,\ell}$, where $\leq_{\mathrm{c}}$ is computable reducibility. Thus bounded Ramsey's theorem for triples is at least as complicated as Ramsey's theorem for pairs. We find that bounded Ramsey's theorem for triples is computationally similar to Ramsey's theorem for pairs despite converse reductions failing. For example, $\forall\ell\,\mathsf{BRT}^3_{2,\ell}$ admits constant-bounded trace avoidance and a weakly low basis, just like $\forall\ell\,\mathsf{RT}^2_\ell$. However, $\mathsf{BRT}^3_{2,4} \not\leq^k_\omega \mathsf{RT}^2_2$ for all k, where $\leq^k_\omega$ is reducibility via the $k+1$ move reduction game of Hirschfeldt and Jockusch. Furthermore, $\mathsf{BRT}^3_{2,4} \not\leq_{\mathrm{c}} \forall\ell\,\mathsf{RT}^2_\ell$.

▸ ELISE PERROTIN, *The logic behind Hanabi.*
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In the card game Hanabi, players must cooperate to play cards in a certain order without seeing their own cards, sharing information through restricted means of communication. This requires epistemic reasoning: players must think about what other players know, don't know, and need to know in order to ensure success. Hanabi has received attention from the learning community as a benchmark to test agents' theory of mind capabilities, but has not been much studied from a symbolic perspective, as formalizing Hanabi in Dynamic Epistemic Logic (DEL), the standard framework for reasoning about knowledge, is at the same time conceptually simple and too heavy for practical use.

The lightweight Epistemic Logic of Observation (EL-O) offers an promising alternative to DEL in which to formalize Hanabi. In particular, states in EL-O are sets of atomic epistemic formulas rather than Kripke models. However, EL-O is not quite rich enough for Hanabi: in particular, it does not handle common knowledge for groups of agents that aren't the coalition of all agents. In this talk I will present my (both published and ongoing) work on enriching EL-O for Hanabi, from determining which standard logic the enriched logic should even be a fragment of, to defining a Hilbert-style axiomatization for this fragment, to then investigating a proof system which allows us to say something about semantics, using it to determine a class of states which includes reachable states in Hanabi and a corresponding class of actions, and finally exploring a number of interesting properties of knowledge in a game of Hanabi. Using these properties, we can then define a lightweight epistemic logic specifically suited for Hanabi, in which states are finite and the satisfiability problem is NP-complete

▸ SIMONE PICENNI, *Conceptions of sets and classes in a class theory with a universal set.*
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Following Forster [2] and Button [1], I consider an iterative construction of the set-theoretic universe in which, at each stage, we form all sets of previously available objects and their complements, yielding a universe with a universal set and self-membered sets. I axiomatise these ideas in a *Forsterian Set Theory* and a *Forsterian Class Theory*, the latter allowing proper classes to be members of sets.

I next revisit the distinction between sets and classes in the context of class theories with a universal set, refining the *indefinite extensibility* account of this distinction [3, 6]. On the proposed picture, concepts that determine sets are those whose extension is not indefinitely extensible *or whose anti-extension* is not indefinitely extensible; concepts whose extension and anti-extension are indefinitely extensible correspond to proper classes. A modal potentialist framework to precisify these ideas is developed along the lines of Studd [6] and Linnebo [4]. I conclude by relating this framework to Schindler's [5] distinction between total and partial classes.

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[5] T. Schindler, *Classes, why and how*, ***Philosophical Studies***, vol. 176 (2019), no. 2, pp. 407–435.

[6] J. P. Studd, ***Everything, More or Less***, Oxford University Press, Oxford, 2019.

▸ PEDRO PINTO, *A finitistic perspective on convergence results.*
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In this talk, I discuss recent work [3] in proof mining concerning the convergence of a generalized viscosity implicit rule in Hadamard spaces, approached from a finitistic perspective. Proof mining [2] employs proof-theoretical techniques to analyse *prima facie* noneffective mathematical proofs with the goal of extracting additional information. Such new information is usually in the form of effective and highly uniform rates or bounds. In the last thirty years, this area of proof theory has been greatly developed by Ulrich Kohlenbach and his collaborators, and proof mining techniques have been particularly successful in applications to nonlinear analysis and adjacent areas. A key feature of proof mining is its ability to unravel complex mathematical arguments, often enabling the replacement of general assumptions by the specific instances actually used in a proof. This has proven particularly fruitful in the lifting of results from linear to nonlinear settings, where many combinatorial aspects of the original arguments can still be exploited. There are, however, limitations and the absence of linear structure often obstructs a direct generalization of such results. In this regard, a finitistic (i.e. quantitative) perspective has proved to be a useful approach for obtaining new results in settings where standard techniques fail, for example in [1, 4].

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[3] Pedro Pinto, *A finitistic approach to a generalized viscosity implicit rule in Hadamard spaces*, in preparation.

[4] Pedro Pinto and Nicholas Pischke, *On the Halpern method with adaptive anchoring parameters*, ***Mathematics of Computation***, to appear.

▸ THOMAS POWELL, *On the structure of convergence proofs in stochastic optimization*.
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The last two years have seen a rapid advance of proof theory being applied in probability and related fields. This broad project includes many case studies in stochastic optimization: Here proof-theoretic insights have led to both new algorithms and new convergence results, where the latter typically come equipped with quantitative information. A byproduct of these efforts has been a deeper understanding of the broad organisational structure of convergence proofs in optimization, many of which hinge on minor variations of a small number of powerful gambits. I will discuss ongoing work (joint with Alex Wan and Benjamin Langton) to devise new, high-level, and domain-specific logical systems that capture these proof techniques, designed for applications in automated reasoning, including the synthesis of efficient and readable convergence rates, and the automated lifting of nonstochastic proofs to the stochastic setting.

▸ IAN PRICE, *Strong reductions are adaptors*.
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Pradic and Price characterise Weihrauch reductions as morphisms in the category of answerable containers over represented spaces [3]. This abstract characterisation allows them to show that much of the algebraic structure of the Weihrauch lattice exists for general reasons rather than being tied to type-2 computability. However, it was not clear how to express strong Weihrauch reductions (reductions where the backwards map only depends on the answers from the oracle and not the initial question) in this setting.

Independently, in an attempt to generalise dependent polynomial functors and optics, Hedges et al. worked on a program of “Fibre Optics” [1], from which they produced a structure they term a “dependent adaptor” [2]. This structure turns out to be exactly what is needed to give a category-theoretic treatment of strong Weihrauch reducibility. We give a characterisation of strong reducibility in terms of adaptors and are able to recover the Galois connection given by cylindrification.

[1] Dylan Braithwaite, Matteo Capucci, Bruno Gavranović, Jules Hedges and Eigil Fjeldgren Rischel, *Fibre Optics*, arXiv.2112.11145 (2021).

[2] Jules Hedges *Fibre Optics II*, draft.

[3] Cécilia Pradic and Ian Price, *Weihrauch Problems as Containers*, ***Crossroads of Computability and Logic: Insights, Inspirations, and Innovations*** (Arnold Beckmann, Isabel Oitaven and Florin Manea, editors), Lecture Notes in Computer Science, vol. 15764, Springer Cham, 2025, pp. 395-409.

▸ MIHAI PRUNESCU, *Closed formulas for arbitrary Kalmar elementary functions.*
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S. Mazzanti proved in 2002 that every Kalmar elementary function can be represented by an arithmetic term over the language consisting of addition $x+y$, modified subtraction $x \dot{-} y$, multiplication xy, quotient $\lfloor x/y \rfloor$, and the exponential function 2^x, see [2]. Moreover, he proved that addition $x+y$, multiplication xy, remainder $x \bmod y$ and the exponential function 2^x already form a basis able to represent any Kalmar elementary function by a closed formula. The proof was barely constructive. Mazzanti has represented some functions which were necessary for the proof, like the greatest common divisor $\gcd(x, y)$ or the 2-adic valuation $\nu_2(x)$, but no further functions were concretely represented during the next 22 years, as this result was not noticed outside the community of computability theorists.

We looked for effective methods to construct arithmetic term representations for functions which are important in number theory and in combinatorics. This research went on three principal directions, which also started to intersect. (1) *Constructions related to Mazzanti's proof.* This approach allowed the construction of closed formulas for some number-theoretic functions like the number of divisors $\tau(n)$, the sum of the divisors $\sigma(n)$, Euler's Totient function $\varphi(n)$, see [5], but also the prime counting function $\pi(n)$ or the n-th prime number $p(n)$, see [7]. Another preprint, see [8], shows how to construct a closed expression $T(n)$ such that for all composite number n, $T(n)$ is a proper divisor of n. (2) *New methods producing much shorter arithmetic term representations.* These efforts produced various general methods to represent all C-recursive functions, see [6, 3, 9], but also multinomial coefficients and related functions, see [11]. A combination of the methods from (1) and (2) produced closed forms able to count all the Mersenne primes, respectively all the Fermat primes less than n, see [4]. (3) *To find a minimal basis for representing Kalmar elementary functions as closed formulas.* It turned out that the three functions: addition $x+y$, remainder $x \bmod y$ and the exponential function 2^x form together a minimal basis able to represent any Kalmar elementary function as an arithmetic term, see [10].

Work in progress [1] bases on a new, general effective method to represent sequences defined by general recurrences

$$x(n+k) = F(n, x(n), \dots, x(n+k-1)),$$

where $F(n, x_0, \dots, x_{k-1})$ is itself an arbitrary arithmetic term, and given initial values $x(0) = y_0, \dots, x(k-1) = y_{k-1}$. The result of the construction is an arithmetic term $f(n, y_0, \dots, y_{n-1})$ such that the identity $x(n) = f(n, y_0, \dots, y_{n-1})$ is true for all $n \in \mathbb{N}$. This construction is further applied to build an arithmetic term which is Turing-complete. This closes the problem of finding a general effective method to represent any Kalmar elementary function by arithmetic terms. Also, it shows that if any function g can be computed in time expressible by a term build up using arithmetic functions and some non-elementary function $a(x)$, for example Ackermann's function, then g itself can be expressed by a term composed by those functions. Further research will concentrate in finding shorter terms for various functions of interest, such that the evaluation of these terms becomes is a feasible task.

[1] Gabriel Istrate, Mihai Prunescu, and Joseph M. Shunia, *Undecidability, Chaos and Universality in Arithmetic Terms*, in preparation.

[2] S. Mazzanti, *Plain Bases for Classes of Primitive Recursive Functions*, ***Mathematical Logic Quarterly***, vol. 48 (2002) no. 1, pp. 93–104.

[3] Mihai Prunescu, *On other two representations of C-recursive integer sequences by terms in modular arithmetic*, ***Journal of Symbolic Computation***, vol. 130 (2025), no. 102433.

[4] ——— *Arithmetic closed forms count the Mersenne primes, the Fermat primes and the twin-prime pairs*, arXiv:2512.01680 (2025).

[5] Mihai Prunescu and Lorenzo Sauras-Altuzarra, *Computational considerations on the representation of number-theoretic functions by arithmetic terms*, ***Journal of Logic and Computation***, vol. 35 (2025), no. 3.

[6] ——— *On the representation of C-recursive integer sequences by arithmetic terms*, ***Journal of Difference Equations and their Application***, vol. 31 (2025), no. 9, pp. 1263–1285.

[7] Mihai Prunescu and Joseph M. Shunia, *On arithmetic terms expressing the prime-counting function and the n-th prime*, arXiv:2412.14594 (2024).

[8] ——— *Elementary closed-forms for non-trivial divisors*, arXiv:2510.26939 (2025).

[9] ——— *On modular representations of C-recursive integer sequences*, ***Journal of Integer Sequences***, vol. 28 (2025), no. 25.5.3.

[10] Mihai Prunescu, Lorenzo Sauras-Altuzarra, Joseph M. Shunia, *A Minimal Substitution Basis for the Kalmar Elementary Functions*, ***Journal of Logic and Computation***, vol. 36 (2026), no. 3.

[11] Lorenzo Sauras-Altuzarra and Joseph M. Shunia, *Arithmetic terms for sums of multinomial coefficients*, ***The Ramanujan Journal***, vol. 68 (2025), no. 93.

▸ GABRIELE PULCINI* AND ACHILLE C. VARZI, *Relative contingency.*
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We address the problem—first raised in [4]—of providing a proof-theoretic characterization of the notion of relative contingency in the context of classical propositional logic, CPL. Let CPL^A be the theory obtained by deductively extending CPL through the addition of a proper propositional axiom A. A formula B is said to be contingent relative to A, or conditionally upon A, just in case B is independent of CPL^A, that is, neither B nor its negation $\neg B$ is provable in CPL^A. To provide a proof-theoretic account of this notion, we introduce, for any truth-functionally contingent formula A, a hypersequent calculus HCC^A whose theorems are exactly the formulas independent of CPL^A. We also show that HCC^A admits an elegant cut-elimination procedure and therefore enjoys the subformula property. These results contribute to the study of refutation calculi [1] and logical complementarity [5] by integrating the proof-theoretic machinery developed in [2] and [3].

[1] V. Goranko, T. Skura and G. Pulcini, *Refutation systems: an overview and some applications to philosophical logics*, ***Knowledge, Proof and Dynamics. The Fourth Asian Workshop on Philosophical Logic*** (F. Liu, H. Ono, J. Yu, editors), Springer, Singapore, 2020, pp. 173–197.

[2] M. Piazza and G. Pulcini, *Uniqueness of axiomatic extensions of cut-free classical propositional logic*, ***Logic Journal of the IGPL***, vol. 24 (2016), no. 5, pp. 708–718.

[3] G. Pulcini and A. C. Varzi, *A hypersequent calculus for classical contingencies*, ***Journal of Philosophical Logic***, vol. 55 (2026), pp. 199–215.

[4] M. Tiomkin, *A sequent calculus for a logic of contingencies*, ***Journal of Applied Logic***, vol. 11 (2013), no. 4, pp. 530–535.

[5] A. C. Varzi, *Complementary sentential logics*, ***Bulletin of the Section of Logic***, vol. 19 (1990), no. 4, pp. 112–116.

▸ GEMMA ROBLES* AND JOSÉ M. MÉNDEZ, *On the existence of 3-valued connexive logics with the variable sharing property.*
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The aim of this paper is to investigate if there are significant 3-valued connexive logics with the variable sharing property (VSP).

Connexive logics are characterized by the presence of some or all of the versions of Aristotle's thesis and Boethius' thesis (cf. [3] and references therein). Let us name a connexive logic "full" it it has all versions of said theses.

On the other hand, the VSP is the property characterizing relevant logics (cf. [2] and references therein). A logic L has the VSP if antecedent and consequent share at least a propositional variable in all L-theorems of implication form.

The motivation behind the paper is that it is to be expected that the connection between antecedent and consequent in valid connexive implications will be reinforced when requiring the fulfillment of the VSP in addition to the validation of the connexive theses.

Now, it is known that there are some (not full) 4-valued connexive logics with the VSP (cf. [1] and references therein), but not (as far as we know) whether there are 3-valued connexive logics with the VSP, be they full or not full. Thus, the aim of this paper is, as said, to investigate this question.

[1] G. Robles, *On relevant acceptable strictly connexive logics*, ***Journal of Logic, Language and Information***, to appear.

[2] S. Standefer, *Variable-sharing as relevance*, ***New directions in relevant logics*** (A. Tedder, I. Sedlar and S. Standefer, editors), Trends in Logic, vol. 63, Springer Cham, 2025, pp. 97–117.

[3] H. Wansing, *Connexive logic*, ***The Stanford Encyclopedia of Philosophy*** (E.N. Zalta and U. Nodelman, editors), Metaphysics Research Lab, Stanford University, 2023.

▸ ANDRÉ ROGNES, *On generalisations of a representation of the integers modulo p, for the purpose of occasionally establishing the unsolvability of diophantine inequalities.*
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It is well known that if a diophantine equation turns out not to have a solution over the integers modulo p, for some natural number p, then it does not have a solution over the standard integers. This is because the integers modulo p are a homomorphic image of the integers. In contrast, the integers modulo p are of little use when faced with diophantine inequalities, as the homomorphic image of the less-than-relation is trivial. The purpose of the present work is to introduce a way of generalising a particular representation of the integers modulo p. The resulting generalisations, believed to be novel, are in the form of decidable Lindenbaum algebras, which we think of as approximations to the Lindenbaum algebra of the first-order theory of integer arithmetic. Crucially if a system of diophantine inequalities turns out not to be solvable in any of the approximations, then it is not solvable over the standard integers. I believe that decision procedures based on the approximations could be implemented and run on personal computers in order to to study diophantine inequalities much as one has studied diophantine equations by interpreting them over the integers modulo p, see e.g. the introductory chapters of the classic book by L. J. Mordell [1].

[1] Louis Mordell, ***Diophantine equations***, Academic press, 1969.

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We revisit and improve our approach [2] of ordinal systems to generate ordinal notation systems based on elementary operations applied strictly from below. This approach first extends the standard ordinal notation system for Γ_0, which is based on Cantor Normal Form and the Veblen function, by extending the Veblen function to Schütte Klammer Symbols [1], which are then nested iteratively. We obtain Veblen terms such as

$$\varphi\left(\begin{array}{ccc} \alpha_1 & \cdots & \alpha_n \\ \begin{pmatrix} \beta_1 & \cdots & \beta_k \\ \gamma_1 & \cdots & \gamma_k \end{pmatrix} & \cdots & \begin{pmatrix} \delta_1 & \cdots & \delta_l \\ \rho_1 & \cdots & \rho_l \end{pmatrix} \end{array}\right)$$

The elementary operations consists of the natural numbers with their standard ordering, disjoint union, lexicographic ordering on pairs, lexicographic ordering on strictly descending finite sequences, and forming subsets of orderings. All these operations preserve well-ordering. By iterating elementary operations, we obtain ordinal notation systems of strength up to ϵ_0.

To go beyond ϵ_0, we form formal terms defined inductively by applying a combination of elementary operations. This yields two orderings on terms: The first one, denoted $\prec'$, is formed directly from the elementary ordering from the ordering induced by the ordering of the underlying components. The other one, denoted by $\prec$ is defined by

$$s \prec t \Leftrightarrow (s \preceq \mathrm{k}(t) \vee (\mathrm{k}(s) \prec t \wedge s \prec' t)$$

This construction reaches, in its limit, the Bachmann-Howard Ordinal.

[1] Kurt Schütte, *Kennzeichnung von Ordnungszahlen durch rekursiv erklärte Funktionen*, ***Mathematische Annalen***, vol. 127 (1954), no. 1, pp. 15–32.

[2] Anton Setzer, *Ordinal systems*, ***Sets and proofs*** (Barry Cooper and John Truss, editors), Cambridge University Press, Cambridge, UK, 1999, pp. 301–331.

▸ ANDREI SIPOŞ, *A worked example in proof mining in uniformly convex Banach spaces.*
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Proof mining is a research paradigm, initiated by G. Kreisel and highly developed by U. Kohlenbach and his students and collaborators, which aims to apply proof-theoretic tools to extract computational content from ordinary proofs in mainstream mathematics (for more information on proof mining, see the book [1] and the recent survey [2]). Usually, the main obstacle that such activity presents is that the arguments frequently used by mathematicians appeal to highly infinitary principles that may not admit a simple computational interpretation. In functional analysis, this would translate into the existence of 'ideal' points such as limits (be they limits superior or inferior). A close viewing of the proof may show that the use of these ideal principles is not really needed for getting to the conclusion of the theorem, at the expense of making the argument slightly messier.

We present a pedagogical worked example that illustrates this methodology. We take a result of Prüss [3] which shows that uniform convexity of Banach spaces is equivalent to a different inequality governed by a modulus. The equivalence proof heavily uses limits superior. We explain how this may be rewritten into an argument that only uses inequalities of norms of ordinary points, and we show how it may be done quantitatively, deriving an expression of said modulus in terms of the modulus of uniform convexity.

[1] U. Kohlenbach, ***Applied proof theory: Proof interpretations and their use in mathematics***, Springer Monographs in Mathematics, Springer-Verlag, 2008.

[2] ——— *Proof-theoretic methods in nonlinear analysis*, ***Proceedings of the International Congress of Mathematicians 2018*** (B. Sirakov, P. Ney de Souza and M. Viana, editors), vol. 2, World Scientific, 2019, pp. 61–82.

[3] Jan Prüss, *A characterization of uniform convexity and applications to accretive operators*, ***Hiroshima Mathematical Journal***, vol. 11 (1981), no. 2, pp. 229–234.

▶ NICHOLAS J.J. SMITH, *On the very idea of formal logical proof.*
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This paper investigates the question: what is a formal logical proof (in general—as opposed to: what is a formal proof in this or that specific proof system)? I begin by setting out a core conception of formal proof and substantiating its designation as 'core'. The heart of this conception is that there must be an effective or mechanical procedure for checking the correctness of a purported proof. I then consider a proposed strengthening of this conception—one that has become standard in the literature on propositional proof complexity—according to which the proof-checking procedure must be able to be carried out in polynomial time. I consider all the arguments in favour of the strengthened conception that I can find or imagine—and reject them all. My conclusion is that, when it comes to analysing the very idea of formal logical proof, we have no good reason to move to the strengthened conception and should stick with the core conception.

▶ GIOVANNI SOLDÀ, *Towards infinitary* GL.
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In this talk, we introduce an infinitary version of the modal logic GL that arises when allowing infintary conjunctions and disjunctions of formulas.

We will then focus on showing that (extensions of) the logic introduced above can arise as provability logics. This will be done as follows: fixing an admissible class A, we consider a theory T extending an infinitary version of KP plus a large chunk of the atomic diagram of A (much in the spirit of what is done in [1]), and show that the provability predicate Pr_T satisfies Gödel's diagonal lemma and the Hilbert-Bernays provability conditions.

This is joint work with Mojtaba Mojtahedi and Fedor Pakhomov.

[1] Jon Barwise, ***Admissible sets and structures***, Perspectives in Mathematical Logic, Springer-Verlag, 1975.

▸ STELLA SPADONI* AND COSIMO PERINI BROGI, *On decidability and bounded proofs in fragments of computability logic.*
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Computability logic (CoL) reinterprets logic as a formal theory of dynamic interaction, modelling statements as computational games between pre-defined agents $\mathcal{M}$ and $\mathcal{E}$ [3]. The present work tackles two open problems regarding CoL fragments CL15 [4] and CL5 [2].

Firstly, we prove CL15 decidable [5]: the potentially infinite search space from resource contraction can be pruned while preserving completeness, bounding contraction applications through a function of the cirquent's complexity.

Secondly, we develop a *novel and purely syntactic* proof that any derivable cirquent in the duplication-free version of CL5 admits a polynomial-size derivation [1], bounded by three structural limits (width, branch length and node size) in the bottom-up proof construction.

[1] Matthew Steven Bauer, *The Computational Complexity of Propositional Cirquent Calculus*, ***Logical Methods in Computer Science***, vol. 11 (2015), no. 1, Art. 12.

[2] Giorgi Japaridze, *Introduction to Cirquent Calculus and Abstract Resource Semantics*, ***Journal of Logic and Computation***, vol. 16 (2006), no. 4, pp. 489–532.

[3] ——— *In the beginning was game semantics* , ***Games: Unifying Logic, Language and Philosophy***, (Ondrej Majer and Ahti-Veikko Pietarinen and Tero Tulenheimo, editors), Logic, Epistemology, and the Unity of Science, vol. 15, Springer, 2009, pp. 249–350.

[4] ——— *The taming of recurrences in computability logic through cirquent calculus, Part I*, ***Archive for Mathematical Logic***, vol. 52 (2013), no. 2, pp. 173–212.

[5] Stella Spadoni, *The Fertile Steppe: Computability Logic and the decidability of one of its fragments*, arXiv:2503.05826v1(2025).

▸ STAN SREDNYAK, *Theory of higher differentiable functionals and applications.*
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Quantum field theory is built on the notion of functionals defined on function spaces. Comparison of theory predictions with experiment requires interpreting data in appropriate functional spaces. We compare physical notions of functionals with mathematical notions of functionals. We then discuss our recent proposal that functionals arbitrarily high in the constructible hierarchy are in fact necessary for description of physical laws. Then we discuss which notions of analysis are necessary to be definable on these higher functional spaces in order for the theory to have predictive power. These notions include the notions of derivative and integral, and we discuss classes of functionals for which these notions can be defined. We provide an example of an infinitely tall tower of functionals which satisfy differential relations at each functional level. We show that if the differential relations have low complexity, then the same is true for the unique solution of these functional relations. This is a paradigmatic example of how higher functionals appear in description of physical systems.

▸ MENGZHOU SUN, *The cohesive and stable Ramsey theorems and proof size over a weak base theory.*
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Over the weak base theory RCA_0^*, most typical Ramsey-theoretic statements are $\forall\Pi_3^0$-conservative, but they differ in terms of the effect on proof size for $\forall\Pi_3^0$ sentences. The usual Ramsey's theorem for pairs and two colours, RT_2^2, induces iterated exponential proof speedup over RCA_0^*, even for very simple sentences [1]. In contrast, Kowalik [2] recently showed that the chain-antichain principle CAC can be polynomially simulated by RCA_0^*. Whether the combinatorial principles discussed above yield significant proof speedup over RCA_0^* depends on the kind of closure properties they imply for the definable cut I_1^0, which is the intersection of all Σ_1^0-definable cuts.

In the present work, we show that $\mathrm{RCA}_0^* + \mathrm{CRT}_2^2$, where CRT_2^2 is the cohesive Ramsey's theorem for pairs, implies exponential closure I_1^0. Consequences include iterated exponential proof speedup of $\mathrm{RCA}_0^* + \mathrm{CRT}_2^2$ over RCA_0^* for Π_1 sentences and the unprovability of CRT_2^2 in $\mathrm{RCA}_0^* + \mathrm{CAC}$. On the other hand, we show that proofs of $\forall\Pi_3^0$ sentences in $\mathrm{RCA}_0^* + \mathrm{SRT}_2^2$, where SRT_2^2 is stable Ramsey's theorem for pairs, can be polynomially simulated by RCA_0^*. Nevertheless, SRT_2^2 also implies a nontrivial property of I_1^0, specifically closure under functions of quasipolynomial growth rate.

Joint work with Leszek Kołodziejczyk.

[1] Leszek Aleksander Kołodziejczyk, Tin Lok Wong, and Keita Yokoyama, *Ramsey's theorem for pairs, collection, and proof size*, ***Journal of Mathematical Logic***, vol. 24 (2024), no. 2, pp.2350007.

[2] Katarzyna W. Kowalik, *A non-speedup result for the chain-antichain principle over a weak base theory*, arXiv:2510.00323.

▸ HSING-CHIEN TSAI, *Atomicity and identity in nontransitive mereology.*
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The so-called nontransitive mereology (NTM) is axiomatized by the following axioms and axiom schema (in the following, $Oxy = \exists z(Pzx \wedge Pzy)$ and $Atom(x) = \forall y(Pyx \rightarrow x = y)$).

(P1: reflexivity) $\forall x Pxx$
(P2: anti-symmetry) $\forall x \forall y((Pxy \wedge Pyx) \rightarrow x = y)$
(SSP: strong supplementation) $\forall x \forall y(\neg Pyx \rightarrow \exists z(Pzy \wedge \neg Ozx))$
(UF: unrestricted fusion principle) $\exists x\alpha(x) \rightarrow \exists z \forall y(Oyz \leftrightarrow \exists x(\alpha(x) \wedge Oyx))$, for any formula $\alpha(x)$ where z and y do not occur free.

There are three versions of atomicity axiom as follows.

(A0) $\forall x \exists y(Atom(y) \wedge Pyx)$
(A1) $\forall x \forall y(Pxy \rightarrow \exists z(Atom(z) \wedge Pzx \wedge Pzy))$
(A2) $\forall x \forall y(Pxy \rightarrow \exists z(Atom(z) \wedge Pzx \wedge \forall z((Atom(z) \wedge Pzx) \rightarrow Pzy))$

A0 is the traditional atomicity axiom and is the weakest version. (It is easy to see that under NTM, A2 implies A1 and A1 implies A0.) We will see that NTM+A2 implies $\forall x \forall y(Pxy \leftrightarrow \forall z((Atom(z) \wedge Pzx) \rightarrow Pzy))$ and that NTM+A1 implies $\forall x \forall y(\forall z((Atom(z) \wedge Pzx) \rightarrow Pzy) \rightarrow Pxy)$. Hence a member's identity is determined by the atoms which it has when the theory considered is either NTM+A1 or NTM+A2. Nonetheless, we will show by defining a complicated model that NTM+A0 fails to guarantee the said proposition.

▸ DIMITRIOS TSINTSILIDAS, *Separation of induction and collection axiom in bounded*

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A classical result of Paris and Kirby [3] in the study of fragments of arithmetic shows that the bounded Σ_{n+1}-collection axiom proves Σ_n-induction and is itself provable using Σ_{n+1}-induction, while the converse fails. Equivalently,

$$\mathsf{I}\Sigma_n \subsetneq \mathsf{B}\Sigma_{n+1} \subsetneq \mathsf{I}\Sigma_{n+1}.$$

In the setting of bounded arithmetic, Buss [1] established an analogous chain of inclusions for polynomial induction and the sharply bounded collection axiom (BB), showing that

$$\mathsf{S}^i_2 \subseteq \mathsf{S}^1_2 + \mathsf{BB}(\Sigma^b_{i+1}) \subseteq \mathsf{S}^{i+1}_2.$$

However, the question of whether these inclusions are strict was left open.

On the other hand, in their influential paper, Krajíček, Pudlák, and Takeuti [2] obtained the first conditional separation in Buss's hierarchy. Assuming the hardness hypothesis $\Sigma^p_{i+1} \not\subseteq \Delta^p_{i+1}/\mathsf{poly}$, they showed that level i of the hierarchy, the theory T^i_2, is strictly weaker than level $i+1$, namely S^{i+1}_2. A key feature of their argument is the extraction from a given proof of an associated interactive computational process, which was later called Student-Teacher games.

In this talk, we study a variant of Student–Teacher games allowing parallel queries, corresponding to theories with the sharply bounded collection axiom. Using this framework, together with lower bounds for such games, we resolve Buss's problem by establishing the strict separations

$$\mathsf{S}^i_2 \subsetneq \mathsf{S}^1_2 + \mathsf{BB}(\Sigma^b_{i+1}) \subsetneq \mathsf{S}^{i+1}_2,$$

under the same hardness assumption $\Sigma^p_{i+1} \not\subseteq \Delta^p_{i+1}/\mathsf{poly}$.

This is joint work with Ondřej Ježil.

[1] Samuel R. Buss, ***Bounded arithmetic***, Princeton University, 1985.

[2] Jan Krajíček, Pavel Pudlák, and Gaisi Takeuti, *Bounded Arithmetic and the Polynomial Hierarchy*, ***Annals of Pure and Applied Logic***, vol. 52 (1991), no. 1-2, pp. 143–153.

[3] Jeff B. Paris and Laurence Kirby, Σ_n*-Collection Schemas in Arithmetic*, ***Studies in logic and the foundations of mathematics***, vol. 96 (1978), pp. 199–209.

► INDIRA TUNGUSHBAYEVA, *On the number of Kaiser-normal classes in a fixed Jonsson spectrum.*
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We work in a countable first-order language L. Let T be an L-theory. T is called a Jonsson theory [1] if T is an inductive theory admitting infinite models, amalgamation and joint embedding properties. For any Jonsson theory T, there exists a semantic model C_T of T [2].

The following definitions were introduced by A. Yeshkeyev.

The Kaiser class [3] of T is a class K_T of L-srtuctures such that $K_T \subseteq Mod(T)$ and for any strucure $M \in K_T$, the deductive closure of $Th_{\forall\exists}(M)$ is a Jonsson theory. We consider a Jonsson spectrum $JSp(K_T)$ [4] of Kaiser class of T, which is defined as follows:

$$JSp(K_T) = \{\Delta \mid \Delta \text{ is a Jonsson theory and } K_T \subseteq Mod(\Delta)\}.$$

Two Jonsson L-theories T_1 and T_2 are called Kaiser-equivalent ($T_1 \triangle T_2$) [3] if $K_{T_1} = K_{T_2}$. It is an equivalence relation.

A Jonsson theory T is called Kaiser-normal if for any $M \in K_T$, $C_{Th_{\forall\exists}(M)}$ is an existentially closed submodel of C_T.

We introduce the relation of Kaiser-equivalence on $JSp(K_T)$. Denote the equivalence class of T by $[T]_\triangle$. $[T]_\triangle$ is called Kaiser-normal if all theories in this class are Kaiser-normal.

There arises a question: how many Kaiser-normal classes are there in $JSp(K_T)$? The following result was obtained.

THEOREM. *For any Jonsson theory T, the following statements hold:*
1) There is at most one Kaiser-normal class in $JSp(K_T)$.
2) If $[T']_\triangle$ is a Kaiser-normal class for some $T' \in JSp(K_T)$, then $T' \triangle T$.

[1] J. BARWISE, ***Handbook of Mathematical Logic***, Nauka, 1982.

[2] A.R. YESHKEYEV, ***Theories and their classes of models. Volume 1***, KarGU, 2024.

[3] A. YESHKEYEV, I. TUNGUSHBAYEVA, AND M. KASSYMETOVA, *On the Kaiser Class of a Jonsson Theory*, ***Symmetry***, vol. 17 (2025), no. 6, pp. 870.

[4] A.R. YESHKEYEV AND O.I. ULBRIKHT, *JSp-cosemanticness and JSB-property of abelian groups*, ***Siberian Electronic Mathematical Reports***, vol. 13 (2016), pp. 861–874.

▸ PATRICK UFTRING, *Independence of premise for IZF*.
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We answer the following question by Frittaion, Nemoto, and Rathjen positively (cf. [3, Problem 1.5]): "Is the following [independence of premise rule] an admissible rule of CZF or any other familiar constructive/intuitionistic set theory T?" To be more precise, we show that whenever IZF proves a statement of the form

$$\neg\psi \to \exists y \varphi(y)$$

such that y does not occur in ψ, then IZF *also* proves

$$\exists y(\neg\psi \to \varphi(y)).$$

Our proof method is a hereditary variant of the famous Friedman-Dragalin A-translation (cf. [1, 2]). Moreover, our argument even extends below to CZF with full separation as well as above to IZF with additional axioms such as Markov's principle. We also discuss why our construction is *not* directly applicable to CZF (with only bounded separation). This project is still work in progress.

[1] A. G. DRAGALIN, *New kinds of realizability and the Markov rule*, ***Doklady Akademii Nauk SSSR***, vol. 251 (1980), no. 3, pp. 534–537.

[2] H. FRIEDMAN, *Classically and intuitionistically provably recursive functions*, ***Higher set theory*** (Oberwolfach, 1977), (G. H. Müller and D. S. Scott), vol. 669, Springer, Berlin, 1978, pp. 21–27.

[3] E. FRITTAION, T. NEMOTO, AND M. RATHJEN, *Choice and independence of premise rules in intuitionistic set theory*, ***Annals of Pure and Applied Logic***, vol. 174 (2023), no. 9, Paper No. 103314, 24.

▸ NESTA VAN DER SCHAAF, *Localic Esakia duality*.
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Classical Priestley duality for distributive lattices and Esakia duality for Heyting algebras use prime ideals to recover their spatial counterpart. This is non-constructive: it uses the Prime Ideal Theorem to recover the lattice. In this talk we outline a constructive Esakia duality.

A constructive Priestley duality was already proven by Townsend [2]. Distributive lattices are replaced by their point-free variant: coherent frames. Priestley spaces are replaced by ordered Stone locales: Stone locales equipped with closed localic partial orders satisfying a point-free Priestley separation axiom. Townsend proved they are dually equivalent.

As point-free ingredients for Esakia duality, we take on the algebra side the recently introduced Heyting frames [1]. On the other hand, observe that Esakia spaces are Priestley spaces whose source map is open. Thus define *Esakia locales* as ordered Stone locales with open source map.

Since localic relations are tricky to handle, we use a third, mediating notion. We prove that closed localic relations with open source map can be recovered by their localic down closure operator. Thus the partial order of an Esakia locale is captured entirely by a join-preserving map on the underlying frame. Axiomatising this structure gives *Esakia frames*.

The conclusion is an equivalence between Heyting frames and Esakia frames, and a dual equivalence between Esakia frames and Esakia locales. Combined, this factorises Townsend's duality, and gives a fully constructive, localic analogue of Esakia duality.

[1] G. Bezhanishvili, L. Carai, and P. J. Morandi, *A frame-theoretic perspective on Esakia duality*, ***Algebra Universalis***, vol. 84 (2023), no. 4, pp. 30.1–30.24.

[2] C. Townsend, *Localic Priestley duality*, ***Journal of Pure and Applied Algebra***, vol. 116 (1997), no. 1, pp. 323–335.

▸ CHARLES WALZER*, ELIO LA ROSA, AND NORBERT GRATZL, *A labelled sequent calculus for truthmaker semantics.*
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This work develops a labelled sequent calculus for logics based on truthmaker semantics, a model-theoretic framework in which the notion of exact truthmaking is primitive. An exact truthmaker for φ is a state that necessitates and is wholly relevant to φ's truth. Different kinds of truthmaking and entailment relations are defined in terms of exact truthmaking and certain closure conditions on models [1]. A core feature of truthmaker semantics is that it provides a framework for constructing hyperintensional logics, i.e., logics that allow for contexts that do not respect classical logical equivalence. Labelled sequent calculi provide a powerful extension of Gentzen-style proof systems in which model-theoretic notions can be represented in a uniform and modular way, while preserving important structural properties from proof theory, such as the syntactic admissibility of cut. Our proof system subsumes previous approaches from Fine and Jago [2] and Korbmacher [4], providing calculi for logics based on non-inclusive, inclusive, and replete classes of exact truthmaker models. The labelling mechanism also allows the calculus to uniformly represent the derivative notions of inexact, loose, and exemplification truthmaking. Different label distributions also allow for the definition of different kinds of entailment relations for each kind of truthmaking, with a proof of soundness and completeness uniformly covering all such cases. Finally, we provide modal companions for each of the above-mentioned variants of truthmaking. This is accomplished by defining a suitable labelled calculus for modal information logics, extending previous results from van Benthem [5] and Knudstorp [3].

[1] K. Fine, *Truthmaker semantics*, ***A Companion to the Philosophy of Language*** (Bob Hale, Crispin Wright, and Alexander Miller, editors), Wiley, Oxford, 2017, pp. 556–577.

[2] K. Fine and M. Jago, *Logic for exact entailment*, ***The Review of Symbolic Logic***, vol. 12 (2019), no. 3, pp. 536–556.

[3] S.B. Knudstorp, *Logics of truthmaker semantics: Comparison, compactness and decidability*, ***Synthese***, vol. 202 (2023), no. 6, pp. 206.

[4] J. Korbmacher, *Proof systems for exact entailment*, ***The Review of Symbolic Logic***, vol. 16 (2022), no. 4, pp. 1260–1295.

[5] J. van Benthem, *Implicit and explicit stances in logic*, ***Journal of Philosophical Logic***, vol. 48 (2019), no. 3, pp. 571–601.

▸ SHUWEI WANG, *Indexed hierarchies and internal structure in the ordinal class in intuitionistic set theories.*
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Classically, the class Ord of all ordinals (i.e. transitive sets with transitive elements) has long served as the natural indices for transfinite hierarchies such as von Neumann's V, Gödel's L or more complicated inner models. In this talk, I will present a distinctive situation in the intuitionistic context, that various hierarchies may induce different optimal indexing classes which are subclasses of Ord.

Take the following abstract recursive construction: fix a definable set-like partial order $\mathcal{R}$ on the class Tr of all transitive sets, which is no coarser than $\subseteq$ yet no finer than $\in$, then the $\mathcal{R}$-*hierarchy* $H_{\mathcal{R}} : V \to \mathrm{Tr}$ can be defined for all set indices $x \in V$ as

$$H_{\mathcal{R}}(x) = \bigcup_{y \in x} P_{\mathcal{R}}(H_{\mathcal{R}}(y)),$$

where $P_{\mathcal{R}}(x) = \{y : y \mathrel{\mathcal{R}} x\}$ is the $\mathcal{R}$-*predecessor* set. When $\mathcal{R} = \subseteq$, $H_{\mathcal{R}}(\alpha)$ is exactly V_α on the von Neumann hierarchy; when $\mathcal{R}$ admits only first-order definable subsets instead, the recursion produces Gödel's constructible universe L.

My talk focuses on the central question: how to produce a *maximal* subclass of V on which $H_{\mathcal{R}}$ is an injection? Classically, Ord is the trivial answer regardless of $\mathcal{R}$, but the proof fails in intuitionistic theories such as IKP or CZF. In this talk, I will present general instructions to construct such a maximal subclass $O_{\mathcal{R}} \subseteq \mathrm{Ord}$ for any (nice enough) specific $\mathcal{R}$. Through realisability arguments, I show it is consistent that hierarchies like V or L yield different indexing classes that are *proper* subclasses of Ord.

The general construction is inspired by Taylor's *plump ordinals* [1], which fits the role of $O_{\mathcal{R}}$ for the V-hierarchy. I will discuss some applications of these ordinal subclasses (where they are more useful than Ord), such as covered in my paper [2].

[1] Paul Taylor, *Intuitionistic sets and ordinals*, ***The Journal of Symbolic Logic***, vol. 61 (1996), no. 3, pp. 705–744.

[2] Shuwei Wang, *Some notes on plump ordinals*, arXiv:2601.23070 (2026).

▸ KAROL WAPNIARSKI, *Leibniz and Russell: on the birth of analytic philosophy.*
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Although Bertrand Russell's interpretation of Leibniz philosophy is as well known as lasting are his contributions to Leibniz scholarship in the 20th century, the question to what extent reading Leibniz influenced development of Russell's own philosophy has received well deserved scholarly attention only recently and remains understudied. While it is usually recognised that Leibniz *did* influenced Russell, the scholarly discourse rarely goes into intricacies of what this influence consisted in precisely and to what extent we should assume its presence and acknowledge its importance. The issue therefore remains obscure, especially when it comes to assessing Leibniz's role in Russell's transition from idealism to logicism.

What I propose is a re-examination of Leibniz's influence on Russell through a study of the so far neglected but important in this regard Russell- Couturat Correspondence [1] (1897-1913), particularly a substantial discussion on Leibniz in the years 1900-1903. What it reveals is that Russell continued to engage with Leibnizian philosophy beyond the composition of his *An Exposition of the Philosophy of Leibniz*, up to the time of writing *The Principles of Mathematics*. It also shows that the beginnings of this engagement, in contrast to the usual account, were not random but carefully though out, and that Russell explicitly recognised himself as a successor of Leibniz's logical endeavours. Through the study of this correspondence, I argue that Leibniz's influence on Russell extends well beyond the documented impact and that reassessing this influence can help us better understand ambitions of the early analytic philosophy.

[1] Bertrand Russell, ***Correspondance sur la philosophie, la logique et la politique avec Louis Couturat (1897–1913)***, ed. by Anne-Françoise Schmid, 2 vols., Kimé, Paris, 2001.

▸ ANDREAS WEIERMANN, *Some averaged zero one laws for segments of proof-theoretic ordinals.*
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We prove some averaged zero laws for ordinals stemming from segments of some prominent proof-theoretic ordinals like ε_0 and Γ_0. The results are based on a mixture of analytic methods and tools from logic. We believe that our results will hold in very general contexts. We also believe that our results will hold for pointwise limits.

We firstly consider zero one laws with respect to the Mahler norm T. Let $T(0) := 0$ and let $T(\alpha) := 2^{T(\alpha_1)} + \cdots + 2^{T(\alpha_k)}$ if $\alpha = \omega^{\alpha_1} + \cdots + \omega^{\alpha_k}$ is in Cantor normal form. We show first (while acknowledging partial support by AI for carrying out some routine steps) as an auxiliary result that there is a real constant C such that for all natural numbers $n > 1$ we have

$$\#\{\alpha < \varepsilon_0 : T(\alpha) = n\} \leq \exp(\exp(C\sqrt{\log(n)})).$$

Since ordinals below ε_0 correspond canonically to rooted non planar trees this result is also of general interest for tree counting since (using the Mahler norm) we obtain a tree counting function which is subexponential.

For a first order sentence φ in the language of linear orders and a natural number $n > 0$ let

$$\delta_{\varepsilon_0}(\varphi)(n) := \frac{\#\{\alpha < \varepsilon_0 : \alpha \models \varphi \wedge T(\alpha) = n\}}{\#\{\alpha < \varepsilon_0 : T(\alpha) = n\}}.$$

Using our auxiliary result we will show that

$$\lim_{x\to\infty} \frac{1}{x} \cdot \sum_{n=1}^{x} \delta_{\varepsilon_0}(\varphi)(n) \in \{0, 1\}.$$

A similar result holds for Γ_0 instead of ε_0. We conjecture that $\lim_{n\to\infty} \delta_{\varepsilon_0}(\varphi)(n) \in \{0, 1\}$.

We secondly consider the standard Gödel coding of ordinals: For $\alpha < \varepsilon_0$ let its Gödel number $G(\alpha)$ be defined as follows. $G(0) = 1$ and if $\alpha = \omega^{\alpha_1} + \cdots + \omega^{\alpha_k}$ is in Cantor normal form then let $G(\alpha) = p_1^{G(\alpha_1)} \cdot \ldots \cdot p_k^{G(\alpha_k)}$ where p_i is the i-th prime number.

Using our first auxiliary result we prove that for all real numbers $x > e^e$ we have

$$\#\{\alpha < \varepsilon_0 : G(\alpha) \leq x\} \leq \exp(\exp(C\sqrt{\log(\log(x))})).$$

For a first sentence φ in the language of linear orders and a real number $t > 1$ let

$$\Delta_{\varepsilon_0}(\varphi)(t) := \frac{\#\{\alpha < \varepsilon_0 : \alpha \models \varphi \wedge G(\alpha) \leq t\}}{\#\{\alpha < \varepsilon_0 : G(\alpha) \leq t\}}.$$

Using our previous results we will show that

$$\lim_{x\to\infty} \frac{1}{\log(x)} \cdot \int_1^x \Delta_{\varepsilon_0}(\varphi)(t)dt/t \in \{0, 1\}.$$

We thirdly consider the following coding of ordinals: For $\alpha < \varepsilon_0$ let its exponential Matula number $M(\alpha)$ be defined as follows. $M(0) = 0$ and if $\alpha = \omega^{\alpha_1} + \cdots + \omega^{\alpha_k}$ is in Cantor normal form then let $M(\alpha) = p_{2^{M(\alpha_1)}} \cdot \ldots \cdot p_{2^{M(\alpha_k)}}$ where p_i is the i-th prime number.

For a first order sentence φ in the language of linear orders and a real number $t > 1$ let

$$\hat{\Delta}_{\varepsilon_0}(\varphi)(t) := \frac{\#\{\alpha < \varepsilon_0 : \alpha \models \varphi \wedge M(\alpha) \leq t\}}{\#\{\alpha < \varepsilon_0 : M(\alpha) \leq t\}}.$$

Refining our previous results we will show that

$$\lim_{x\to\infty} \frac{1}{\log(x)} \cdot \int_1^x \hat{\Delta}_{\varepsilon_0}(\varphi)(t)dt/t \in \{0, 1\}.$$

Our results can be extended to Γ_0 instead of ε_0. We conjecture that $\lim_{t\to\infty} \Delta_{\varepsilon_0}(\varphi)(t) \in \{0, 1\}$ and $\lim_{t\to\infty} \hat{\Delta}_{\varepsilon_0}(\varphi)(t) \in \{0, 1\}$. We also believe that limit laws will hold for monadic second order sentences from the theory of linear orders.

Our results here differ from previous results from [1] where we were able to show limit laws (but not zero one laws) for ε_0 with respect to the standard norm function or the Matula coding of ordinals.

[1] A. Weiermann, A.R. Woods, *Some natural zero one laws for ordinals below* ε_0, ***How the World Computes (CiE 2012)***, (S. Barry Cooper, Anuj Dawar, and Benedict Löwe, editors), Lecture Notes in Computer Science, vol 7318, Springer, Berlin, 2012, 723–732.

▸ GUOHUA WU* AND JIA LI ZHENG, *On a subclass of Baire Class α Functions*.
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Semmes introduced the Tree Game, $G_T(f)$, to characterise the class of Borel functions $f : \omega^\omega \to \omega^\omega$. Subsequently, Nobrega provided a characterisation for subclasses of the Borel functions which can be decomposed into Baire-α functions on $\Pi^0_{\alpha+1}$-partitions by analysing Player *II*'s winning strategies. In this talk, we will continue the research along this line and show that f is Baire-α on a $\Pi^0_{\alpha+1}$-partition if and only if $f = \lim_{n\to\infty} f_n$, the pointwise limit of (f_n), where each f_n is Baire-α and for all x, for almost all k, $f_k(x) = f(x)$.

▸ J. ALBERT YOO, *Non-composability of realised epistemic access: a type-theoretic account of temporal integration via parametricity*.
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Classical epistemic logic idealises knowledge as closed under conjunction. That picture is too strong for cognitively realised knowledge: an agent may separately retrieve A and separately retrieve B without having A and B jointly available as one epistemic unit, because the whole must be formed within a limited microtemporal window of temporal integration or summation. This motivates a typed split between ideal knowledge-that and a realised-access layer closer to episodic retrieval, perceptual binding, and embodied performance. I formalise this claim in a two-layer type theory. A cartesian stable layer represents ideal propositional content and preserves ordinary closure under conjunction. A substructural realised layer, indexed by agent and microtemporal window, represents temporally situated retrieval and binding. In that realised layer, integrated access $A \star B$ cannot be formed from separate realised derivations of A and B alone; formation requires an explicit same-window binding witness. The main result is a parametric no-go theorem: there is no closed polymorphic term that, uniformly in agents, windows, stable contexts, realised contexts, and content types, maps derivations of $RDeriv_{i,w}(\Gamma, \Delta, A)$ and $RDeriv_{i,w}(\Gamma, \Delta, B)$ to a derivation of $RDeriv_{i,w}(\Gamma, \Delta, A \star B)$ unless a binding witness is present. More precisely, the realised layer validates elimination from $A \star B$ back to its constituents but has no unrestricted introduction rule from separate retrieval judgments, and the parametric theorem shows that no closed polymorphic term can recover such a rule. The resulting picture preserves classical closure for ideal knowledge while locating failure of composition precisely at the formation of temporally integrated epistemic wholes.

Abstracts of talks presented by title