

2026 SPRING MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The Palmer House, Chicago, IL
Central APA Meeting
February 18–21, 2026

Program committee: Ali Enayat, Joel David Hamkins (chair), and Snow Zhang.

The 2026 Spring Meeting is part of the meeting of the Central Division of the American Philosophical Association. All ASL meeting participants must register for the APA conference at <https://www.apaonline.org/mpage/2026central>.

The Central APA Meeting runs February 18–21, 2026 and includes other talks and sessions of interest to logicians. The complete program is available on the website of the Central Division of the APA. Please see the APA program for room numbers.

Program change: Unfortunately, James Walsh (New York University) is unable to deliver his invited talk *Modal definability in Kripke's theory of truth*. Joel Hamkins will speak in his place, delivering the talk *Mathematicians do not agree on the essential structure of the complex numbers*.

Wednesday February 18, 6:00 PM–7:50 PM

ASL Invited Talks Session I

Chair: TBA

6:00 – 6:50 **Francesca Zaffora Blando** (Carnegie Mellon University) *Schnorr randomness and rates of convergence to the truth*.

7:00 – 7:50 **Joel David Hamkins** (University of Notre Dame) *Mathematicians do not agree on the essential structure of the complex numbers*.

Wednesday February 18 6, 8:00 PM–9:20 PM

ASL Contributed Talks Session

Chair: TBA

8:00 – 8:20 **Sara Ayhan** (Ruhr University) *Contradictions without negation and a proof-theoretic, bilateralist account of connexive logics*.

8:30 – 8:50 **Erin Carmody** (SUNY Delhi) *Mitchell rank for supercompactness*.

9:00 – 9:20 **John Marvin** (University of Chicago) *Why did Ed Nelson think arithmetic is inconsistent, and how did he try to prove it?*

Thursday February 19, 7:00 PM–8:50 PM

ASL Invited Talks Session II

Chair: TBA

7:00 – 7:50 **Øystein Linnebo** (Oslo University) *What is potentialism?*

8:00 – 8:50 **Jason Zesheng Chen** (University of California Irvine) *The emergence of a foundational role*.

Friday February 20, 7:00 PM–8:50 PM

ASL Invited Talks Session III

Chair: TBA

7:00 – 7:50 **Neil Barton** (National University Singapore) *Metasemantics and the continuum hypothesis.*
8:00 – 8:50 **Mateusz Lelyk** (University of Warsaw) *Around the categoricity of Peano Arithmetic.*

Abstracts of invited plenary lectures

► NEIL ALEXANDER BARTON, *Metasemantics and the Continuum Hypothesis.*

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The Continuum Hypothesis featured top of Hilbert's list of 23 problems in 1900. Today, we still consider the question, with various programmes pulling in different directions. This conceptual diversity raises a puzzle: In what sense do we *disagree* when we talk about it? A standard assumption takes it that the content of our thought about classes and the Continuum Hypothesis is uniform across agents and times. Assuming a moderate view of how content is determined, I reject this assumption. However, I also argue that whilst the Continuum Hypothesis can have different content for different agents, it can also be determinate for certain programmes. In particular, I suggest that there is a fault line between those who think there are uncountable sets and recent countabilist programmes.

► JASON ZESHENG CHEN, *The emergence of a foundational role.*

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This talk will sketch the emergence of a particular purpose served by set theory, and then attempt to characterize it as foundational, perhaps with outlooks similar to those characterized in Maddy's insightful analysis of discourse regarding foundations ([1, 2]).

The narrative will revolve around descriptive set theory and the technical branch of set theory known today as Borel equivalence relations theory (*cf.* [3, 4]). I am going to show that, throughout its development, the theory has come to serve a peculiar foundational purpose - which I shall call **Barrier Exposure**. To illustrate, I return to the early days of descriptive set theory, when the Borel sets were first introduced in [5]. By studying the introduction of Borel sets in its historical context, I argue that they were introduced as a way to restrict attention to the tractable problems in analysis. This point remains salient (although not always explicit) in the later development of (descriptive) set theory amidst controversies surrounding the axiom of choice, for instance in [6], where abstract equivalence relations were first considered. Particular attention will be paid, in this case, to the common context and motivations of Borel and Luzin, and subsequently to the more modern development in the technical literature, e.g., [7, 8], ultimately culminating in the methodological maxim: *heed where intractability begins*.

From this perspective, set theory can be seen to play a two-fold role: one of organizing and relating various structures from diverse fields of mathematics and their attendant classification problems (akin to model theory and category theory's Productive Guidance [9]), and the other of delineating the boundaries of the tractable and intractable such problems.

I will argue that this is a role that can be considered foundational. More precisely, I

will attempt to come to a conditional conclusion: this role is as foundational as some of the other candidates considered in the current foundational literature ([9, 10, 11]). In other words, I will argue that insofar as category theory and model theory can be said to play a foundational role in mathematics, as evidenced in its providing the kinds of services outlined by Maddy and Baldwin, the same kind of role is being played by set theory today.

- [1] PENELOPE MADDY, *Set-theoretic foundations*, *Contemporary Mathematics* (Andrés Caicedo, James Cummings, Peter Koellner, and Paul Larson, eds.), vol. 690, American Mathematical Society, Providence, Rhode Island, 2017, pp. 289–322.
- [2] PENELOPE MADDY, *What do we want a foundation to do?*, *Reflections on the Foundations of Mathematics: Univalent Foundations, Set Theory and General Thoughts* (Stefania Centrone, Deborah Kant, and Deniz Sarikaya, eds.), Springer International Publishing, Cham, 2019, pp. 293–311.
- [3] GREG HJORTH, *Borel equivalence relations*, *Handbook of Set Theory* (Matthew Foreman and Akihiro Kanamori, eds.), Springer Netherlands, Dordrecht, 2010, pp. 297–332.
- [4] SU GAO, *Invariant descriptive set theory*, Chapman and Hall/CRC, 2008.
- [5] ÉMILE BOREL, *Leçons sur la théorie des fonctions*, vol. 1, Gauthier-Villars, 1898.
- [6] NIKOLAI NIKOLAEVICH LUZIN, *Sur les ensembles analytiques*, *Fundamenta Mathematicae*, vol. 10 (1927), pp. 1–95.
- [7] MATTHEW FOREMAN, DANIEL RUDOLPH, AND BENJAMIN WEISS, *The conjugacy problem in ergodic theory*, *Annals of Mathematics*, vol. 173 (2011), no. 3, pp. 1529–1586.
- [8] LUCA MOTTO ROS, *Classification problems from the descriptive set theoretical perspective*, *Research Trends in Contemporary Logic*, (Melvin Fitting, Dov Gabbay, Massoud Pourmahdian, Adrian Rezus, and Ali Sadegh Daghighi, eds.), Forthcoming.
- [9] JOHN T. BALDWIN, *Exploring the generous arena*, *The Philosophy of Penelope Maddy* (Sophia Arbeiter and Juliette Kennedy, eds.), vol. 31, Springer International Publishing, Cham, 2024, pp. 143–164.
- [10] TIM BUTTON, SEAN WALSH, AND WILFRID HODGES, *Philosophy and model theory*, Oxford University Press, Oxford, 2018.
- [11] JOHN T. BALDWIN, *Model theory and the philosophy of mathematical practice: formalization without foundationalism*, Cambridge University Press, New York, NY, 2018.

- JOEL DAVID HAMKINS, *Mathematicians do not agree on the essential structure of the complex numbers*.

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What is the essential structure of the complex numbers? Mathematicians, it turns out, do not generally agree—indeed one can find sharply worded disagreements. Do we have a purely algebraic conception of the complex numbers, taking it as an algebraically closed field with only its algebraic structure? Or do we have an analytic view, as a field over the real numbers, distinguished as a particular subfield? Or should we have a topological view? Perhaps we have a rigid conception of the complex plane, with the coordinate structure of the real and imaginary parts. Many mathematicians find it fundamentally wrong to break the symmetry between i and $-i$, and indeed the various perspectives give rise to fundamentally different understandings of the automorphism group, and they are not all fully bi-interpretable nor even mutually interpretable. I

shall place the whole discussion into the context of the philosophy of structuralism and the question of what is a number. This talk is based on my essay at <https://www.ininitelymore.xyz/p/complex-numbers-essential-structure>.

- MATEUSZ LELYK, *Around the categoricity of Peano Arithmetic.*

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The goal of the talk is to present some recent developments revolving around the question: how different may various structures satisfying the axioms of arithmetic be? By the "axioms of arithmetic" we mean the usual basic axioms for addition and multiplication of natural numbers and the scheme of induction - the system typically called *Peano Arithmetic*. The talk is divided into three parts. In the first one, we explain a recent result obtained jointly with David Gonzalez, Dino Rossegger and Patryk Szlufik which provides a full classification of possible Scott complexities of countable models of Peano Arithmetic, [3]. Intuitively, Scott complexity analysis delivers quantitative information about how hard it is to describe a given structure uniquely up to an isomorphism. More precisely, a countable structure \mathcal{M} has Scott complexity Γ iff there is an $\mathcal{L}_{\omega_1, \omega}$ sentence ϕ of complexity Γ such that (1) $\mathcal{M} \models \phi$, (2) any countable model \mathcal{N} which satisfies ϕ is isomorphic to \mathcal{M} and (3) ϕ is "the least complicated" sentence satisfying (1) and (2). The results of [3] show that, except for a few exceptions, any infinite realizable Scott complexity is a Scott complexity of a model of Peano Arithmetic.

In the second part of the talk the above result is contrasted with the situation for Peano's axioms in the context of full second-order semantics. As famously shown by Dedekind, each two models of Peano's axioms, whose *all* subsets satisfy the induction scheme, are isomorphic. Over recent years there were some attempts, most notably in [5], [4] and [2] to obtain first-order versions of the Dedekind categoricity argument. We argue that these arguments are best seen as being about concrete schemes, as opposed to theories and present a convenient framework for stating this kind of first-order internal categoricity arguments which explicitly tracks how strong means are required to establish the definiteness claim for a given scheme. Finally, we comment on internally categorical schemes which axiomatize a proper subtheory of Peano Arithmetic.

The third part of the talk is devoted to the notion of *solidity*: a categoricity-like notion for first-order theories introduced in [1] and further studied in [2]. Roughly speaking, to verify that a theory T is solid, we fix an arbitrary model $\mathcal{M} \models T$ and look at the class of models of T which are interpreted in \mathcal{M} . Then we narrow this class to these models which are rich enough to interpret back an isomorphic copy of \mathcal{M} (and the isomorphism is \mathcal{M} -definable). A theory T is solid if, for an arbitrarily chosen \mathcal{M} , this last class contains uniquely models which are (\mathcal{M} -verifiably) isomorphic to \mathcal{M} . In the talk we explain why this can be seen as a meaningful categoricity-like notion for first-order theories, give a proof of solidity of Peano Arithmetic and comment on the possibility of constructing its proper solid subtheories.

Part two is based on a joint work with Piotr Gruza, while part three is a joint work with Piotr Gruza and Leszek Kołodziejczyk.

[1] ALI ENAYAT, *Variations on a Visserian theme, A tribute to Albert Visser*, (J. van Eijk, R. Iemhoff, J. Joosten, editors), College Publications, London, 2016, pp. 99–110.

[2] ALI ENAYAT, AND MATEUSZ LELYK, *Categoricity-like properties in the first-order realm, Journal for the Philosophy of Mathematics*, vol. 1 (2024), pp. 63–98.

[3] DAVID GONZALEZ, MATEUSZ LELYK, DINO ROSSEGGER, AND PATRYK SZLUFIK, *Classifying the complexity of models of arithmetic*, arXiv:2507.12025.

[4] PENELOPE MADDY AND JOUKO VÄÄNÄNEN, *Philosophical Uses of Categoricity Arguments*, Cambridge University Press, 2023.

[5] JOUKO VÄÄNÄNEN, *Tracing internal categoricity*, *Theoria. A Swedish Journal of Philosophy*, vol. 87 (2021), no. 4, pp. 986–1000.

► ØYSTEIN LINNEBO, *What is potentialism?*

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Aristotle famously claimed that the only coherent form of infinity is potential, not actual. However many objects there are, it is possible for there to be yet more; but it is impossible for there in fact to be infinitely many objects. Although this view was superseded by Cantor’s transfinite set theory, even Cantor regarded the collection of all sets as “unfinished” or incapable of “being together”. In recent years, there has been a revival of interest in *potentialist* approaches to the philosophy and foundations of mathematics, which invoke collections that are merely potential [3], [5], [4]. Here I attempt to clarify what potentialism is and is not, and to respond to some objections.

The heart of potentialism, I argue, is the phenomenon of *incompleteness*, understood as the claim that there are collections whose members cannot all be “jointly available”. Although this incompleteness has a very natural modal analysis, the use of modality is shown to be inessential. To be jointly available is to be a plurality (as in plural logic [1]). This enables a non-modal analysis of potentialism based on a “critical” plural logic, which rejects the unrestricted plural comprehension scheme that allows any formula to define a plurality [2].

Next, these analyses are used to tackle some central objections to potentialism. Some find the modality used in potentialism problematic. I canvas some ways to understand the modality but observe that, thanks to the non-modal analysis of incompleteness, the modality can also be understood as playing a merely heuristic role. Others contend that potentialism is ultimately equivalent to actualism. I respond that the restricted plural logic at the core of potentialism has sharp, logical-mathematical consequences concerning classes and perhaps also concerning the logic of quantification.

[1] GEORGE BOOLOS, *To be is to be a value of a variable (or to be some values of some variables)*, *Journal of Philosophy*, vol. 81 (1984), no. 8, pp. 430–449.

[2] SALVATORE FLORIO AND ØYSTEIN LINNEBO, *The Many and the One: A Philosophical Study of Plural Logic*, Oxford University Press, 2021.

[3] ØYSTEIN LINNEBO, *The potential hierarchy of sets*, *The Review of Symbolic Logic*, vol. 6 (2013), no. 2, pp. 205–228.

[4] CHRIS SCAMBLER, *Can all things be counted?*, *Journal of Philosophical Logic*, vol. 42 (2021), no. 5, pp. 1079–1106.

[5] JAMES STUDD, *The iterative conception of set: A (bi-)modal axiomatisation*, *Journal of Philosophical Logic*, vol. 42 (2013), no. 5, pp. 697–725.

► JAMES WALSH, *Modal definability in Kripke’s theory of truth*.

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In *Outline of a Theory of Truth*, Kripke introduces some of the central concepts of the logical study of truth and paradox. He informally defines some of these—such as groundedness and paradoxicality—using modal locutions. We introduce a modal language for regimenting these informal definitions. Though groundedness and paradoxicality are expressible in the modal language, we prove that intrisicality—which Kripke emphasizes but does not define modally—is not. This follows from a characterization of the modally definable relations and an attendant axiomatization of the modal semantics.

- FRANCESCA ZAFFORA BLANDO, *Schnorr randomness and rates of convergence to the truth.*

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Lévy's Upward Theorem—a fundamental martingale convergence theorem that is also a cornerstone of Bayesian epistemology and philosophy of science—establishes that the conditional expectation of an integrable random variable converges almost surely to the random variable's true value with increasing information. In this talk, I will show that, by appealing to computability theory and the theory of algorithmic randomness, one can characterize the probability-one set of points along which convergence to the truth obtains and identify the conditions under which convergence to the truth occurs at a computable rate. I will focus on two main results featuring Schnorr randomness, a canonical algorithmic randomness notion that has turned out to play an important role in computable analysis and measure theory. We will see that, for a natural class of effective random variables—the class of lower semi-computable random variables with a computable p -norm (where $p \geq 1$ is itself a computable real number)—the Schnorr random points coincide with the truth-conducive points. Then, we will see that, for the same class of effective random variables, (i) the Schnorr random points weakly compute a rate of convergence for the conditional expectation and (ii) convergence to the truth is guaranteed to occur at a computable rate along all Schnorr random points of computably dominated degree. I will discuss these results, as well as their philosophical ramifications, in the general setting of computable Polish spaces equipped with computable probability measures: a setting that provides an appropriately broad class of spaces and measures to develop Bayesian epistemology for computationally bounded probabilistic reasoners. These results are joint work with Simon Huttegger (UC Irvine) and Sean Walsh (UCLA).

Abstract of Contributed Talks

- SARA AYHAN, *Contradictions without negation and a proof-theoretic, bilateralist account of connexive logics.*

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The study of connexive logics has undergone a recent increase in attention. A usual conception of these contra-classical systems is that (among further conditions) they validate theorems called *Aristotle's Theses* and *Boethius' Theses*, which are non-theorems of classical logic. A standard example for a formulation of Aristotle's Thesis is the following: $\sim(\sim A \rightarrow A)$, expressing that no formula is implied by its own negation. A prominent system among these is Wansing's (2005) logic C , which is also a non-trivial negation-inconsistent logic (also called *contradictory logics*) in the sense that it contains formulas of the form A and $\sim A$ among its theorems. Thus, both the conception of connexive and of contradictory logics seem to rely heavily on the presence of a negation.

I will show, though, that this is not necessarily the case. Therefore, I will present and discuss the negation-free fragment of a bilateralist version of C . With this approach, firstly, a different notion of contradictory logics that does not rely on negation inconsistency but rather on a primitive notion of refutation will be motivated. Secondly, I will propose a change of the usual definition commonly given for connexive logics. Though there are different proposals for definitions of (different forms of) connexive

logics, what they all have in common is that they focus on whether or not certain formulas are validated. My definition will radically break with these definitions by purely relying on a rule-based account of connexivity. Thus, I will propose a conception of connexive logics based on (bilateralist) proof-theoretic semantics.

[1] HEINRICH WANSING, *Connexive modal logic*, *Advances in Modal Logic, Vol. 5* (Renate Schmidt and Ian Pratt-Hartmann and Mark Reynolds and Heinrich Wansing, editors), College Publications, London, 2005, pp. 367–383.

- ERIN CARMODY, *Mitchell rank for supercompactness*.
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Almost every large cardinal studied so far comes with degrees of its existence, or seems to have the potential for degrees. For example, Mitchell rank for measurable cardinals provides degrees for measurable cardinals. In a previous paper, I showed how to precisely define the degrees of inaccessible cardinals as Mahlo did for Mahlo cardinals. In another paper, Gitman and Habić define degrees of Ramsey cardinals. This paper defines a Mitchell rank for supercompact cardinals. Along with the degrees of a large cardinal comes the forcings which can kill a large cardinal to any desired degree. It is not yet known if for every degree of a large cardinal there is a forcing to kill it so that in the forcing extension it has the desired maximum rank or degree, but so far they seem to go hand in hand. In this paper, I will also show how to force to cut down a measurable cardinal's Mitchell rank to any (possible) desired rank, and then we will see how to softly kill Mitchell rank for supercompactness. In the end, we will include similar results for other large cardinals including interactions between supercompact and strongly compact cardinals. Before we see the measurable and supercompact cases, I will state several theorems which fit into the killing-them-softly theme of forcing to destroy a large cardinal property while preserving lesser large cardinal properties. For example, if κ is any degree t of inaccessibility there is a forcing extension where κ is still t -inaccessible but not $t+1$ -inaccessible. In this theorem t is a meta-ordinal since there are more than Ord (using Ω for the order type of Ord to define the meta-ordinals) many degrees of inaccessible cardinals. A meta-ordinal is a formal syntactic expression for the order-types of Ord and beyond. We have a similar theorem for Mahlo cardinals, and so on.

[1] ARTHUR APTER AND MOTI GITIK, *The least measurable can be strongly compact and indestructible*. *The Journal of Symbolic Logic*, vol. 63 (1998), no. 4, pp. 1404–1412.

[2] ERIN CARMODY, *Killing them softly: degrees of inaccessible and Mahlo cardinals*. *Mathematical Logic Quarterly*, vol. 63 (2017), no. 3–4, pp. 256–264.

[3] ERIN CARMODY, VICTORIA GITMAN, AND MIHA E. HABIĆ, *A Mitchell-like order for Ramsey and Ramsey-like cardinals*. *Fundamenta Mathematicae*, vol. 248 (2020), pp. 1–32.

[4] WILLIAM B. EASTON, *Powers of regular cardinals*. *Annals of Mathematical Logic*, vol. 1 (1970), pp. 139–178.

[5] HAIM GAIFMAN, *A generalization of Mahlo's method for obtaining large cardinal numbers*. *Israel Journal of Mathematics*, vol. 5 (1967), pp. 188–200.

[6] VICTORIA GITMAN, *Ramsey-like cardinals*. *The Journal of Symbolic Logic*, vol. 76 (2011), no. 2, pp. 519–540.

[7] JOEL DAVID HAMKINS, *Destruction or preservation: as you like it*. *Annals of Pure and Applied Logic*, vol. 91 (1998), no. 2–3, pp. 191–229.

[8] ————— *Extensions with the approximation and cover properties have no new large cardinals*. *Fundamenta Mathematicae*, vol. 180 (2003), no. 3, pp. 257–277.

[9] ——— *A multiverse perspective on the axiom of constructibility*. *Infinity and Truth*, vol. 25 (2013).

[10] JOEL DAVID HAMKINS AND SAHARON SHELAH, *Superdestructibility: a dual to Laver's indestructibility*. *The Journal of Symbolic Logic*, vol. 63 (1998), no. 2, pp. 549–554.

[11] THOMAS JECH, *Set Theory*, Third Millennium Edition, Springer, 2002.

[12] R. BJÖRN JENSEN AND KENNETH KUNEN, *Some combinatorial properties of L and V*. Unpublished, 1969.

[13] AKIHIRO KANAMORI, *The Higher Infinity*, Second Edition, Springer, 2003.

[14] MENACHEM MAGIDOR, *How large is the first strongly compact cardinal? Or a study on identity crises*. *Annals of Mathematical Logic*, vol. 10 (1976), pp. 33–57.

[15] PAUL MAHLO, *Über lineare transfinite Mengen*, *Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, Mathematisch-Physische Klasse*, vol. 63 (1911), pp. 187–225.

[16] WILLIAM MITCHELL, *Hypermeasurable cardinals*. *Logic Colloquium '78* (Mons), (Maurice Boffa, Dirk van Dalen, and Kenneth McAlloon, editors), North Holland, 1979, pp. 303–316.

[17] JASON A. SCHANKER, *Weakly measurable cardinals*. *Mathematical Logic Quarterly*, vol. 57 (2011), no. 3, pp. 266–280.

► JOHN MARVIN, *Why did Ed Nelson think arithmetic is inconsistent, and how did he try to prove it?*

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In 2012, luminary Princeton mathematician Ed Nelson infamously announced to the Foundations of Mathematics email list that he had a strategy for proving that Primitive Recursive Arithmetic (PRA) is inconsistent, and that thus all stronger foundational systems (e.g. PA, ZFC) including it are too. This talk will exhaustively examine Nelson's writings and endeavor to explain how and why he came to believe that arithmetic is inconsistent, explaining his radically skeptical “strict formalism” and the intellectual-historical conditions to which it responds, looking at his unusual understanding of Thomist and Aristotelian philosophical convictions, and detailing the religious obsessions and even experiences that prompted his quixotic project. It will also sketch the technical ideas behind his unsuccessful proof and its development of Chaitin's work and the Kritchman-Raz “Surprise Examination Paradox” version of the second incompleteness theorem (as well as his little-discussed original ideas toward the same goal), detailing in turn where Nelson's ultimate strategy is irreparably flawed. Finally, this talk will discuss Nelson's constructive vision for mathematics and the surprisingly prescient essential role he saw for computer proof systems, as early as the 1990s, in a post-inconsistency mathematical discipline.

Abstracts Presented By Title

► JOACHIM MUELLER-THEYNS, *Stäbner's stone*.
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We analyzed a certain paradox in a sober manner through the observation that from somehow evident: *If x is all-mighty, then x can even create a stone that is so heavy that x can not carry it*, and tacit: *If x is all-mighty, x can carry all*, it can be proven by contradiction that *nothing is all-mighty*.

We then obtained the same theorem within a considerably simpler and less special mathematical theory from single: *If x is all-mighty, x can even create something with the empty property.*

I. Undefined predicates are $AllM$ (“all-mighty”) and CC (“can create”).

DEFINITION. (i) $x \text{CC}_X y : \leftrightarrow x \text{CC} y \wedge Xy$ (x can create y with property X);

(ii) $\text{CCS}_X x : \leftrightarrow \exists y x \text{CC}_X y$ (x can create something with X);

(iii) $\text{CCSAP} x : \leftrightarrow \forall X \text{CCS}_X x$ (x can create something for and with any property).

BUCHHOLZ’S AXIOM. $AllM \sqsubseteq \text{CCSAP}$, videlicet $\forall x (AllM x \rightarrow \text{CCSAP} x)$.

Note: We had previously defined and interposed *all-creativity*: $AllC x : \leftrightarrow \forall y x \text{CC} y$; $AllM \sqsubseteq AllC$, $AllC \sqsubseteq \text{CCSAP}$.

II. We define $S_x y : \leftrightarrow \text{Stone} y \wedge y \text{IsSoHeavyThat} \neg x \text{CanCarry} y$, $\text{CanCarryAll} x : \leftrightarrow \forall y x \text{CanCarry} y$, and use $AllM \sqsubseteq \text{CanCarryAll}$ as further axiom.

THEOREM. $\neg \exists x AllM x$.

Proof (by contradiction). Suppose $AllM x_0$ for some x_0 . By axiom I, $\text{CCSAP} x_0$. Hence $\forall X \text{CCS}_X x_0$, whence, particularly, $\text{CCS}_{S_{x_0}} x_0$. So $x_0 \text{CC}_{S_{x_0}} y_0$ for some y_0 , viz. $x_0 \text{CC} y_0 \wedge S_{x_0} y_0$. By $S_{x_0} y_0$, $y_0 \text{IsSoHeavyThat} \neg x_0 \text{CanCarry} y_0$, whence $\neg x_0 \text{CanCarry} y_0$. However, by $AllM x_0$ and $AllM \sqsubseteq \text{CanCarryAll}$, $\text{CanCarryAll} x_0$, whence $x_0 \text{CanCarry} y_0$. Thus $x_0 \text{CanCarry} y_0 \nleftrightarrow x_0 \text{CanCarry} y_0$.

III. We return to level I. By using the empty property \emptyset instead of S_x , we already obtain that *nothing is all-mighty*:

THEOREM. $\neg \exists x AllM x$.

Proof. As in proof II, we get $\forall X \text{CCS}_X x_0$, from which now $\text{CCS}_\emptyset x_0$. So $x_0 \text{CC}_\emptyset y_0$, viz. $x_0 \text{CC} y_0 \wedge \emptyset y_0$, whence $\emptyset y_0$. However, since $\forall x \neg \emptyset x$, $\neg \emptyset y_0$.

IV. We add the predicate *God* and obtain that *there is no all-mighty God*:

COROLLARY. $\neg \exists x (AllM x \wedge \text{God} x)$.

Proof. Suppose $AllM x_0 \wedge \text{God} x_0$, whence $AllM x_0$. However, from theorem III (or II), $\forall x \neg AllM x$, whence $\neg AllM x_0$. Contradiction!

V. The factual use of “all” (as in science) is literal (cf. “all primes are odd”). Nonetheless, at least colloquially, ‘all’ is often used in the sense of almost all, most, or even many. This might be reflected in the spelling “almighty” (cf. *großmächtig*).

By admitting non-empty properties only, we may have found a *logical* loophole:

DEFINITION. $\text{CCSARP} x : \leftrightarrow \forall X (\text{Real} X \rightarrow \text{CCS}_X x)$ (x can create something for and with any real property), where $\text{Real} X : \leftrightarrow \exists x Xx$.

Thanks: Wilfried Buchholz, Klaus-Peter Stäbner, Shannon Miller; LARA webinar & LUW, Jean-Yves Béziau; Lem, Dostoevsky; Ulrike Hahn, Andreas Haltenhoff, Thomas De Motter; and ‘Peana Pesen’.