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2026 WINTER MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Washington DC
2026 Joint Mathematics Meeting
January 4–7, 2026

Program Committee: Uri Andrews (chair), Alexi Block Gorman, and Assaf Shani.

This program is a draft for the 2026 Winter Meeting of the ASL to be held within the 2026 Joint Mathematics Meeting, January 4–7, 2026 at the Walter E. Washington Convention Center in Washington DC. For updated registration, travel, housing, and program information, please see the JMM website `https://jointmathematicsmeetings.org/jmm`.

In addition to the talks list below, the ASL Special Session on *Computability and Its Applications* organized by Wesley Calvert, Johanna Franklin, and Valentina Harizanov will be held on Monday January 5, 8:00am–11:50am and 1:00pm–4:50pm in Room 103A of the Walter E. Washington Convention Center.

SUNDAY, JANUARY 4
Walter E. Washington Convention Center, Room 103A

Morning

9:00 – 10:00	ASL Tutorial: **Deirdre Haskell** (McMaster), *TBA, part I.*

Afternoon

1:00 – 2:00	ASL Tutorial: **Deirdre Haskell** (McMaster), *TBA, part II.*

TUESDAY, JANUARY 6
Walter E. Washington Convention Center, Room 103A

Morning

9:00 – 9:50	Invited Lecture: **Jennifer Pi** (Oxford), *Model theory of operator algebras.*
10:00 – 10:50	Invited Lecture: **Garrett Ervin** (Cal Tech), *New arithmetic laws for order types.*

Afternoon

1:00 – 1:50	Invited Lecture: **Jenna Zomback** (Amherst), *Ergodic theorems, weak mixing, and chaining.*
2:00 – 2:50	Invited Lecture: **Filippo Calderoni** (Rutgers), *The isomorphism problem for finitely generated orderable groups.*
3:00 – 5:00	Contributed Talks: *see page 2.*

WEDNESDAY, JANUARY 7
Walter E. Washington Convention Center, Room 103A

Morning

9:00 – 9:50	Invited Lecture: **Damir Dzhafarov** (Connecticut), *From topology to combinatorics, via computability theory.*
10:00 – 10:50	Invited Lecture: **Rahim Moosa** (Waterloo), *Recent advances in the model theory of algebraic dynamics.*

Afternoon

1:00 – 1:50	Invited Lecture: **Joe Miller** (Wisconsin), *Beyond the Turing degrees.*

CONTRIBUTED TALKS
Walter E. Washington Convention Center, Room 103A

Contributed Talks, Tuesday January 6

3:00 – 3:20	**Corrie Ingall,** (University of Connecticut), *The reverse mathematics of the Hahn-Mazurkiewicz theorem.*
3:30 – 3:50	**Heidi Benham,** (University of Connecticut), *Problem reducibility of a weakened version of the Ginsburg-Sands theorem.*
4:00 – 4:20	**Hyung Mook Kang,** (North Texas University), *Continuous hyperfiniteness.*
4:30 – 4:50	**Shay Logan,** (Kansas State University), *Hyperformal restrictions of classical logic won't be traditional relevance logics.*
5:00 – 5:20	**Athar Abdul-Quader,** (Purchase College), *The lattice problem for models of* PA.
5:30 – 5:50	**Brian Wynne,** (Lehman College), *Existentially closed Archimedean lattice ordered groups with strong unit.*

Abstracts of invited plenary lectures

▸ FILIPPO CALDERONI, *The isomorphism problem for finitely generated orderable groups.*
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In this talk we discuss how mathematical logic, and in particular descriptive set theory, provides a uniform way to analyze classification problems. We outline how this machinery can be employed to analyze orderable groups. Finally we present some ongoing results suggesting that the classification for finitely generated orderable groups might be as complicated as it can be. This is joint work with Adam Clay.

▸ DAMIR DZHAFAROV, *From topology to combinatorics, via computability theory.*
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Combinatorics has played an important role in the history of reverse mathematics, with a particular focus on Ramsey's theorem. In this talk, I will survey some recent developments in this investigation, but with a twist. Namely, I will focus on recent joint work with Benham, DeLapo, Solomon, and Villano exploring the proof-theoretic and computability-theoretic strength of a theorem not in combinatorics, but in topology. In their 1979 paper "Minimal Infinite Topological Spaces," Ginsburg and Sands proved that every infinite topological space has an infinite subspace homeomorphic to exactly one of the following five topologies on ω: indiscrete, discrete, initial segment, final segment, and cofinite. The original proof of the Ginsburg-Sands theorem features an interesting application of Ramsey's theorem for pairs (RT^2_2). We analyze it using Dorais's formalization of CSC spaces. Among our results are that the Ginsberg-Sands theorems for CSC spaces is equivalent to ACA_0, while for Hausdorff spaces it is provable in RCA_0. Furthermore, if we enrich a CSC space by adding the closure operator on points, then the Ginsburg-Sands theorem turns out to be equivalent to the Chain-Antichain Principle, an easy corollary of RT^2_2. The most surprising case is that of the Ginsburg-Sands theorem restricted to T_1 spaces. Here, we show that the principle lies strictly between ACA_0 and RT^2_2, yielding perhaps the first natural theorem of ordinary mathematics (i.e. from outside of logic) to occupy this interval. I will discuss some of the techniques used in obtaining these results, as well as some ideas for further research.

▸ GARRETT ERVIN, *New arithmetic laws for order types.*
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Let LO denote the class of linear orders. Given two orders $A, B \in \mathsf{LO}$, their sum $A + B$ is the order obtained by placing a copy of B to the right of A. This operation generalizes the familiar sum for ordinals.

Despite the rich diversity of linear order types, arithmetic in $(\mathsf{LO}, +)$ is surprisingly nice in certain respects: Lindenbaum showed that $(\mathsf{LO}, +)$ satisfies a completely general Euclidean division theorem, and Aronszajn found an elegant structural characterization of the additively commuting pairs of linear orders. Yet although these theorems generalize basic facts about sums of natural numbers, the published proofs are somewhat difficult and ad hoc, and do not transparently connect arithmetic in $(\mathsf{LO}, +)$ to arithmetic in a "numerical" setting like $(\mathbb{N}, +)$ or $(\mathbb{R}, +)$.

In recent joint work with Eric Paul, we develop a systematic approach to the arithmetic of $(\mathsf{LO}, +)$ by studying groups of order-automorphisms of infinite $\mathbb{Z}$-sums of linear orders. Using this approach, we give new, unified proofs of Lindenbaum's and Aronszajn's theorems. Our proofs give significantly more structural information than the originals, and show that division and commutativity in $(\mathsf{LO}, +)$ are closely connected to arithmetic in the ordered group of real numbers.

We then generalize this approach to semigroups acting by convex embeddings on one-sided infinite sums of linear orders, obtain an arithmetic characterization of commutativity in $(\mathsf{LO}, +)$, and determine which commutative semigroups can be represented in $(\mathsf{LO}, +)$. In this talk I will give an overview of our work, outline some of the proofs, and discuss future directions.

▸ JOSEPH S. MILLER, *Beyond the Turing degrees.*
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The Turing degrees measure the computability-theoretic complexity of elements of Cantor space (i.e., infinite binary sequences). We can code other (countably supported) mathematical objects as binary sequences, so one might hope to use the Turing degrees to measure their complexity as well. However, in the absence of a reasonably "canonical" coding, this approach fails. For example, in 2004, I proved that not all continuous functions on the unit interval have Turing degree. This was, by no means, the first such example. However, it turns out that a slight extension of the Turing degrees is sufficient to measure the complexity of points in $\mathcal{C}[0,1]$, and in fact, in any (effective) Polish space. I will discuss this and other extensions of the Turing degrees that arise in effective mathematics.

▸ RAHIM MOOSA, *Recent advances in the model theory of algebraic dynamics.*
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The model theory of algebraic differential equations, via the first order theory of differentially closed fields, is a well-established subject that continues to generate significant applications. There has been an analogous model theory for algebraic **difference** equations since the work of Chatzidakis and Hrushovski in the early nineteen nineties on the theory of algebraically closed fields equipped with a generic automorphism (ACFA). While there have been a number of striking applications of algebraic dynamics based on ACFA, by Chatzidakis–Hrushovski and Medvedev–Scanlon for example, the subject remains less well developed than its differential counterpart. This is partly due to the fact that ACFA does not admit quantifier elimination and is not stable. However, the quantifier-free fragment of ACFA is stable, and already captures much of algebraic dynamics. I will report on recent and ongoing work with Moshe Kamensky on a systematic development of geometric stability theory for a certain finite-dimensional quantifier-free part of ACFA, with applications to algebraic dynamics.

▸ JENNIFER PI, *Model theory of operator algebras.*
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Model theory has seen great success when applied to other fields of mathematics, especially with regard to algebraic and geometric questions. On the other hand, applications of model theory have also led to further model-theoretic concepts to study. In this talk, I discuss some recent advances in the interchange of ideas between logic and operator algebras.

Operator algebras are certain collections of bounded operators on Hilbert spaces. They give a strong foundation for understanding quantum mechanics, as well as non-commutative flavors of geometry, topology, and probability theory. A huge theme within operator algebras (and indeed in all mathematics) is that of classification, which asks the broad question of how we can tell objects apart or conclude they are the same. I will discuss some work related to classification problems in operator algebras, and logical variants thereof. This is largely based on joint work with Michał Szachniewicz and Mira Tartarotti.

▸ JENNA ZOMBACK, *Ergodic theorems, weak mixing, and chaining.*
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Several recent methods for proving pointwise ergodic theorems for probability measure preserving actions of free groups critically use weak mixing properties of Markov measures on the boundary of the free group. However, it was previously unknown which Markov measures are weak mixing. In joint work with Anush Tserunyan, we give a complete characterization of such measures. It turns out that, under mild non-degeneracy assumptions, they are exactly the Markov measures arising from strictly irreducible transition matrices — a condition introduced by Bufetov in 2000 for a different purpose. The proof of this characterization goes through proving equivalences with a new dynamical condition on the action that we call chaining, which is interesting in its own right.

Abstracts of contributed talks

▸ ATHAR ABDUL-QUADER, *The lattice problem for models of* PA.
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The lattice problem for models of PA is to determine which lattices can be represented either as lattices of elementary substructures of a model of PA or, more generally, which can be represented as lattices of elementary substructures of a model N that contain a given elementary substructure M of N. In this talk, we will focus on one particular example that we hope motivates the main technique for realizing finite lattices as interstructure lattices, due to Schmerl in 1986 [2]. This technique involves representations of lattices as lattices of equivalence relations and the combinatorial properties of such representations. This talk is based on our recent survey, with Roman Kossak, of the lattice problem, its history, and this technique in [1].

[1] Abdul-Quader, Athar and Kossak, Roman, *The Lattice Problem For Models of PA*, ***The Bulletin of Symbolic Logic***, Published online (2025), pp. 1–30.

[2] James H. Schmerl, *Substructure lattices of models of Peano arithmetic*, ***Logic Colloquium '84*** (J.B. Paris and A.J. Wilkie and G.M. Wilmers, editors), Elsevier, North-Holland, Amsterdam, 1986, pp. 225–243.

▸ HEIDI BENHAM, *Problem reducibility of a weakened version of the Ginsburg-Sands theorem.*
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A recent paper by Benham, DeLapo, Dzhafarov, Solomon, and Villano analyzes the computability theoretical and reverse mathematical strength of a topological theorem by Ginsburg and Sands, along with that of weakened versions. The original theorem states that every infinite topological space has an infinite subspace homeomorphic to one of the following on the natural numbers: indiscrete, initial segment, final segment, discrete, or cofinite. In this original paper, it is claimed that the theorem is a consequence of Ramsey's theorem for pairs, and though it has been shown by Benham, et al. that the full theorem is equivalent over RCA_0 to ACA_0, there is a weakened version that is equivalent over RCA_0 to CAC, a consequence of RT^2_2. One interesting feature of the proof of this equivalence is that, not only an application of CAC, but also an application of ADS, which is a consequence of CAC, is used. This inspires the question of whether this weakened version of the Ginsburg-Sands theorem and CAC, when viewed as Weihrauch problems, are equivalent.

I will present some new progress that has been made on this question. This progress involves developing several new combinatorial problems related to CAC and ADS, one of which is Weihrauch equivalent to the weakened version of the Ginsburg-Sands theorem, and showing a variety of reducibilities between them.

▸ CORRIE INGALL, *The reverse mathematics of the Hahn-Mazurkiewicz theorem.*
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The Hahn-Mazurkiewicz theorem is an early result in geometric analysis that has not yet been studied within the context of reverse mathematics. In pursuit of understanding the reverse mathematical strength of the entire theorem, we will discuss the forward implication, which states that if $f : [0,1] \to \mathbb{R}^d$ is continuous, then range$f$ is compact, connected, and locally connected. This forward implication, despite its relative simplicity in standard mathematics, still requires careful consideration within reverse mathematics. We will discuss some recent results concerning the properties of these range sets, in particular how both defining and utilizing local connectedness in this setting presents interesting challenges.

▸ HYUNG MOOK KANG* AND STEPHEN JACKSON, *Continuous hyperfiniteness.*
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It has been a long-lasting problem, posed by Weiss (1984), whether any Borel action of a countable amenable group on a standard Borel space gives rise to a hyperfinite Borel equivalence relation. It is possible to consider these questions in a continuous setting to see whether the induced orbit equivalence relation of a continuous action is continuously hyperfinite, liminf of G-clopen finite equivalence relations, continuously embeddable to E_0 or continuously reducible to E_0. We discuss that the one implies the other, but not necessarily the other way around. Also, we discuss the continuous analogue of the Borel asymptotic dimension, discuss its consequences, including embeddability.

▸ SHAY LOGAN, *Hyperformal restrictions of classical logic won't be traditional relevance logics.*
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Let $\mathcal{L}$ be a language with connectives $\{c_i\}_{i\in I}$ each with finite arity a_i and let the set of position-labels, **Lab**, be $\{\langle c_i, n\rangle \mid i \in I \text{ and } 1 \leq n \leq a_i\}^*$. So **Lab** is the set of all finite sequences of pairs where each pair consists of a connective c_i and a choice of an argument position in c_i. A *matrix* for $\mathcal{L}$ is a pair $\langle SV, D\rangle$ where $D \subseteq SV$ together with functions $e_i : SV^{a_i} \to SV$ for each $i \in I$. Intuitively, SV is a set of semantic values, D is the designated values, and e_i is the 'truth table' for connective c_i.

Given a matrix M for $\mathcal{L}$, an M-hypersemantics is a set V of functions $\mathsf{At}\times\mathbf{Lab} \to SV$. Each $f \in V$ extends to a function $f^+ : \mathcal{L} \times \mathbf{Lab} \to SV$ by saying $f^+(p, \overline{\mathtt{x}}) = f(p, \overline{\mathtt{x}})$ for $p \in \mathsf{At}$ and $f^+(c_i(B_1, \ldots, B_{a_i}), \overline{\mathtt{x}}) = e_i(f^+(B_1, \langle c_i, 1\rangle\overline{\mathtt{x}}), \ldots, f^+(B_{a_i}, \langle c_i, a_i\rangle\overline{\mathtt{x}}))$. The logic defined by a hypersemantics is $\{A \mid \text{for all} f \in V, f^+(A) \in D\}$.

In the remainder we focus on the case where $SV = \{0,1\}$, $D = \{1\}$ and V satisfies the following Independence Assumption: for all f, all $p \neq q$ in At, and all $\overline{\mathtt{y}}$, there is g so that $g(p, \overline{\mathtt{x}}) = f(p, \overline{\mathtt{x}})$ for all $\overline{\mathtt{x}}$, but $g(q, \overline{\mathtt{y}}) \neq f(q, \overline{\mathtt{y}})$. We say that this g agrees with f on q. We will also focus on two connectives on which we suppose our hypersemantics behaves loosely classically. We will call these connectives '$\to$' and '$\otimes$' and write them with infix notation. With these conventions in hand, the 'broadly classical' behavior we assume is that $e_\to(x, y) = \sup(1 - x, y)$ and $e_\otimes(x, y) = \inf(x, y)$. Finally, we also suppose that for all $f \in V$ we have that $f^+(p \to (q \to (p \otimes q)), \varepsilon) = 1$, that $f^+((q \to p) \to (q \to p), \varepsilon) = 1$ and that $f^+((q \to (p \otimes r)) \to (q \to (p \otimes r)), \varepsilon) = 1$.

In this talk we will show that the logic defined by *any* hypersemantics satisfying these assumptions will not be contained in any of the traditional relevance logics. Concretely, what we will prove is the following.

Theorem. *For all $f \in V$, $f^+((q \to (p \otimes r)) \to (q \to (p \otimes q)), \varepsilon) = 1$.*

This leaves us with very little hope of defining any of the traditional relevance logics by means of hyperformal restrictions of classical logic, something that much of the author's recent past work seemed to suggest was possible.

▸ BRIAN WYNNE, *Existentially closed Archimedean lattice ordered groups with strong unit*.
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A lattice-order group, or ℓ-group, is a group equipped with a partial ordering, invariant under translation on the left or right, that is a lattice ordering, so an ℓ-group may be viewed as a structure for the first-order language $\mathcal{L} = \{+, -, 0, \sqcap, \sqcup\}$ (here $\sqcap$, $\sqcup$ are binary function symbols for the lattice operations of meet and join, respectively). An ℓ-group $\mathcal{G}$ is Archimedean if $\mathcal{G} \models \forall x \geq 0\, \forall y \geq 0, [(\bigwedge_{n=1}^{\infty} nx \leq y) \Rightarrow x = 0]$, and the underlying group of an Archimedean ℓ-group is always Abelian. If $\mathcal{G}$ is an ℓ-group, and if $u \in \mathcal{G}$, then u is a strong unit in $\mathcal{G}$ if $\mathcal{G} \models u \geq 0 \wedge \forall x \geq 0 (\bigvee_{n=1}^{\infty} x \leq nu)$. Let $\mathbf{W}^+$ be the category whose objects are the non-zero Archimedean ℓ-groups with distinguished strong unit, viewed as structures for the first-order language $\mathcal{L}_1 = \{+, -, 0, \sqcap, \sqcup, 1\}$ (here the constant symbol 1 is interpreted as the distinguished strong unit), and whose morphisms are the unit-preserving ℓ-group homomorphisms (i.e. the $\mathcal{L}_1$-homomorphisms). A prototypical example of a $\mathbf{W}^+$-object is $C(X)$, the continuous real-valued functions on a compact Hausdorff space X with pointwise operations and the constant function with value 1 as the distinguished strong unit. In fact, the Yosida representation theorem says that every $\mathbf{W}^+$-object is isomorphic to a sub-object of some such $C(X)$. If $\mathcal{G}$ is a $\mathbf{W}^+$-object, then $\mathcal{G}$ is existentially closed (e.c.) in $\mathbf{W}^+$ if any existential $\mathcal{L}_1$-formula with parameters from $\mathcal{G}$ that is satisfied in some extension of $\mathcal{G}$ in $\mathbf{W}^+$ is satisfied already in $\mathcal{G}$. I will discuss what is known so far about the e.c. $\mathbf{W}^+$-objects, including axiomatizations, examples, and the existence of e.c. prime-model extensions.

Abstracts of talks presented by title

▸ JOACHIM MUELLER-THEYS, *Abstract Concept Logic*.
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I. *Intension.* Let $\mathcal{B} \neq \emptyset$ be some set, $B, C, \ldots \in \mathcal{B}$ (concepts, *Begriffe*). Further, $\sqsubseteq_{\text{int}} \subseteq \mathcal{B} \times \mathcal{B}$ be some reflexive and transitive relation on $\mathcal{B}$ (intensional sub-concept). $B \sqsupseteq_{\text{int}} C :\Leftrightarrow C \sqsubseteq_{\text{int}} B$ (intensional super-concept, attribute); $B \equiv_{\text{int}} C :\Leftrightarrow B \sqsubseteq_{\text{int}} C$ & $B \sqsupseteq_{\text{int}} C$ (intensional equivalence).

According to tradition, *content* is "sum" of attributes: $\iota(B) := \{C \in \mathcal{B} \colon C \sqsupseteq_{\text{int}} B\}$. We can now *compare concepts by content*: $B \subseteq C :\Leftrightarrow \iota(B) \subseteq \iota(C)$; $B \supseteq C :\Leftrightarrow C \subseteq B$ $\Leftrightarrow \iota(B) \supseteq \iota(C)$; $B \doteq C :\Leftrightarrow B \subseteq C$ & $B \supseteq C \Leftrightarrow \iota(B) = \iota(C)$ (less, more, equal).

We proved the theorem that *more-content coincides with intensional sub-concept*: $B \supseteq C \Leftrightarrow B \sqsubseteq_{\text{int}} C$, which we call *intensional reciprocity.* As corollaries, $B \subseteq C \Leftrightarrow$ $B \sqsupseteq_{\text{int}} C$; $B \doteq C \Leftrightarrow B \equiv_{\text{int}} C$.

II. *Extension.* Let $\mathcal{B}$ be as in I; $G \neq \emptyset$, $g, \ldots \in G$(*Gegenstände*); and $Z \subseteq \mathcal{B} \times G$. $B(g) :\Leftrightarrow B\, Z\, g$ (applies, *trifft zu*). Likewise, $g\, I\, B :\Leftrightarrow B(g)$ (has) (*logical incidence*).

We define *(extensional) subconcept*: $B \sqsubseteq C :\Leftrightarrow \forall g \in G\, \big(B(g) \Rightarrow C(g)\big)$, whence $B \sqsupseteq C$, $B \equiv C$. $\sqsubseteq$ is reflexive and transitive.

Obviously, $\varepsilon(B) := \{g \in G \colon B(g)\}$ is the *extent* of B. We can now *compare concepts by extent*: $B \preceq C :\Leftrightarrow \varepsilon(B) \subseteq \varepsilon(C)$, whence $\succeq$, $\simeq$.

Usually, there is *(conceptual) polynomy*, viz. there are $B, C \in \mathcal{B}$ such that $B \simeq C$, but $B \neq C$. Thereby, $\preceq$ is not antisymmetric, and ε is not injective.

Less-extent coincides with sub-concept: $B \preceq C \Leftrightarrow B \sqsubseteq C$, which we call *extensional* *proportionality*. Thereby $\succeq\, = \,\sqsupseteq$, $\simeq\, = \,\equiv$.

We introduced *extensional* content: $\iota_{\text{ext}}(B) := \{C : C \sqsupseteq B\}$, whereby $\subseteq_{\text{ext}}$, $\supseteq_{\text{ext}}$, $\bar{=}_{\text{ext}}$; $\supseteq_{\text{ext}}\, = \,\sqsubseteq$, $\subseteq_{\text{ext}}\, = \,\sqsupseteq$, $\bar{=}_{\text{ext}}\, = \,\equiv$ (analogous to I). We now obtain $B \supseteq_{\text{ext}} C \Leftrightarrow B \preceq C$, whence $\subseteq_{\text{ext}}\, = \,\succeq$, $\bar{\equiv}_{\text{ext}}\, = \,\simeq$.

III. *Relating Intension and Extension.* The famous *Law of Reciprocity* asserts that more-content and less-extent coincide. By $\supseteq\, = \,\sqsubseteq_{\text{int}}$ and $\sqsubseteq\, = \,\preceq$, the *Reciprocity Theorem* $B \supseteq C \Rightarrow B \preceq C$ is equivalent to *extensional soundness*, videlicet $B \sqsubseteq_{\text{int}} C \Rightarrow B \sqsubseteq C$. However, for the same reason, $B \supseteq C \Leftrightarrow B \preceq C$ is true only if $\sqsubseteq_{\text{int}}$ is *extensional*, viz. $\sqsubseteq_{\text{int}}\, = \,\sqsubseteq$.

Genesis. My interest arose with Bertram Kienzle's seminar on Frege's "Grundlagen" (1990-1), followed by a paper with proven *Extensional Law of Reciprocity* (cf. II). In subsequent decades, we developed *Basic Concept Logic* (TU Dresden 2013; "Counterexamples to FCA: $A' \neq B$", BSL 21, 2 (2015), 232-3; "Basic Concept Logic", XVII SLALM 2017, BSL 24, 3 (2018), 370-1), *First-order Concept Logic* (BSL 27, 1 (2021), 96), and, recently, *ACL* (among other issues stimulated through LUW, Anca Pascu et. al., April 2025). This would have been impossible without Wilfried Buchholz. We thank Andreas Haltenhoff, Heinrich Wansing, Uwe Scheffler, Gerhard Schönrich, Dresden logic students; Ulrich Kohlenbach; Jean-Yves Béziau; and 'Peana Pesen'.