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2025 NORTH AMERICAN ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

New Mexico State University
Las Cruces, NM, USA

May 13-16, 2025

Please note: This program is a draft for the upcoming ASL North American meeting and is subject the change. Updated versions will be posted on the ASL website and the conference website.

Program Committee: Uri Andrews, Valeria de Paiva, Ilya Shapirovsky, Caroline Terry, and Simon Thomas (chair).
Local Organizing Committee: John Harding (chair), Andre Kornell, Joel Lucero-Bryan, Patric Morandi, Bruce Olderding, Ilya Shapirovsky, and Son Tran.
Please see `https://math.nmsu.edu/asl-2025/index.html` for additional information.

The conference will take place in Science Hall, home of the Mathematics Department at New Mexico State University. The plenary lectures and tutorials will take place in Science Hall 102. The special sessions and contributed talks will be held in Science Hall 106, 107, 108, 109 and 115. The welcoming reception will be held at 6pm on Tuesday May 13 at the Courtyard by Marriott.

TUESDAY, May 13

Morning

8:30 – 9:00	Registration.
9:00 – 10:00	Opening Remarks, followed by Invited Lecture: **Maryanthe Malliaris** (University of Chicago), *New perspectives on model-theoretic complexity*
10:00 – 10:20	Coffee.
10:30 – 11:20	Tutorial Lecture 1: **Mariya I. Soskova** (University of Wisconsin), *Enumeration reducibility and effective mathematics, part 1.*
11:30 – 12:20	Invited Lecture: **Patrick Lutz** (UC Berkeley), *Measure hyperfiniteness and lossless expansion.*

Afternoon

2:00 – 3:50	Special Session CT1, ML1, MT1 and ST1. See pages 3–6.
4:00 – 4:20	Coffee.
4:30 – 5:20	Invited Lecture: **Sergei Artemov** (Graduate Center CUNY), *Consistency of* PA *is a serial property, and it is provable in* PA*.*
6:00 – 9:00	Welcoming Reception at the Courtyard by Marriott.

WEDNESDAY, May 14

Morning

8:30 – 9:00	Registration.
9:00 – 9:50	Tutorial Lecture 1: **Julia Wolf** (University of Cambridge), *An introduction to higher-order stability, part 1.*
10:00 – 10:20	Registration and coffee.
10:30 – 11:20	Tutorial Lecture 2: **Mariya I. Soskova** (University of Wisconsin), *Enumeration reducibility and effective mathematics, part 2.*
11:30 – 12:20	Invited Lecture: **Benjamin Castle** (University of Illinois), *Reconstruction problems in geometry via model theory.*

Afternoon

2:00 – 3:50	Special Session CL1, CT2, ML2, MT2 and ST2. See pages 3–6.
4:00 – 4:20	Coffee.
4:30 – 5:50	Contributed Talks. See pages 6–6.

THURSDAY, May 15

Morning

8:30 – 9:00	Registration.
9:00 – 9:50	Tutorial Lecture 2: **Julia Wolf** (University of Cambridge), *An introduction to higher-order stability, part 2.*
10:00 – 10:20	Coffee.
10:30 – 11:20	Tutorial Lecture 3: **Mariya I. Soskova** (University of Wisconsin), *Enumeration reducibility and effective mathematics, part 3.*
11:30 – 12:20	Invited Lecture: **Alejandro Poveda** (Harvard University), *Recent developments on the theory of supercompact cardinals.*

Afternoon

12:40 – 4:10	Special Session PA1. See pages 3–6.
2:00 – 3:50	Special Session CL2, CT3 and MT3. See pages 3–6.
4:00 – 4:20	Coffee.
4:30 – 5:20	Invited Lecture: **Felix Weilacher** (UC Berkeley), *Separating complexity classes of LCL problems on grids.*

FRIDAY, May 16

Morning

8:30 – 9:00	Registration.
9:00 – 9:50	Tutorial Lecture 3: **Julia Wolf** (University of Cambridge), *An introduction to higher-order stability, part 3.*
10:00 – 10:20	Coffee.
10:30 – 12:20	Special Session CL3, ML3, PA2 and ST3. See pages 3–6.

SPECIAL SESSIONS

CL. Combinatorics and Logic

(Organized by Artem Chernikov, Damir Dzhafarov, and Andrew Marks)

Session CL1: Wednesday, May 14, Science Hall 109.

2:00 – 2:30 **Saugata Basu** (Purdue University), *Cohomological VC-density: bounds and applications.*

2:40 – 3:10 **Chris Conidis** (College of Staten Island), *The computability of the uniform Krull intersection theorem.*

3:20 – 3:50 **Cecelia Higgins** (UC Los Angeles), *Complexity of finite Borel asymptotic dimension.*

Session CL2: Thursday, May 15, Science Hall 109.

2:00 – 2:30 **David Gonzalez** (UC Berkeley), *Ramsey theoretic statements motivated by generic computability.*

2:40 – 3:10 **Sam Murray** (McGill University), *Borel fractional perfect matchings in quasi-transitive amenable graphs.*

3:20 – 3:50 **Rehana Patel** (Wesleyan University), *Countable infinitary theories admitting an invariant measure.*

Session CL3: Friday, May 16, Science Hall 109.

10:30 – 11:00 **Theodore A. Slaman** (UC Berkeley), *Cauchy subsequences and the cohesiveness principle.*

11:10 – 11:40 **Atticus Stonestrom** (University of Notre Dame), *An arithmetic algebraic regularity lemma.*

11:50 – 12:20 **Andy Zucker** (University of Waterloo), *Enumerations of Boolean algebras.*

CT. Computability Theory

(Organized by Meng-Che (Turbo) Ho and Manlio Valenti)

Session CT1: Tuesday, May 13, Science Hall 106.

2:00 – 2:50 **Julia Knight** (University of Notre Dame), *Computable Π_2 Scott sentences.*

3:00 – 3:20 **Keshav Srinivasan** (George Washington University), *Effective ultrapowers of algebraic extensions of $\mathbb{Q}$.*

3:30 – 3:50 **Andrew DeLapo** (University of Connecticut), *Computability and countable second-countable spaces.*

Session CT2: Wednesday, May 14, Science Hall 106.

2:00 – 2:50 **Reed Solomon** (University of Connecticut), *Strong indivisibility for graphs.*

3:00 – 3:20 **Steffen Lempp** (University of Wisconsin), *Chains and antichains in the Weihrauch lattice.*

3:30 – 3:50 **Ang Li** (University of Wisconsin), *Countable ordered groups and Weihrauch reducibility.*

Session CT3: Thursday, May 15, Science Hall 106.

2:00 – 2:50 **Alexander Melnikov** (Victoria University, Wellington), *The space of continuous functions: a playground for logic.*

3:00 – 3:20 **Java Darleen Villano** (University of Connecticut), *Computable categoricity relative to a generic degree.*

3:30 – 3:50 **Jacob Fiedler** (University of Wisconsin), *Extending affine subspaces in higher dimensions.*

ML. Modal Logic

(Organized by Wesley Holliday and Ilya Shapirovsky)

Session ML1: Tuesday, May 13, Science Hall 115.

2:00 – 2:20 **Joel Lucero-Bryan** (New Mexico State University), *On modal logics arising from the Čech-Stone compactification of ordinals.*

2:30 – 2:50 **Vladislav Sliusarev** (New Mexico State University), *Criteria of local tabularity of products of modal logics.*

3:00 – 3:20 **Tadeusz Litak** (FAU Erlangen-Nürnberg), *TBA.*

3:30 – 3:50 **Fedor Pakhomov** (University of Ghent), *The logic of correct models.*

Session ML2: Wednesday, May 14, Science Hall 115.

2:00 – 2:20 **Andrew Bacon** (University of Southern California), *What is the logic of logical necessity?*

2:30 – 2:50 **Ahmee Christensen** (UC Berkeley), *First-order Fischer Servi logic.*

3:00 – 3:20 **Joseph McDonald** (University of Alberta) *Monadic ortholattices: completions and duality*

3:30 – 3:50 **Eric Pacuit** (University of Maryland), *Common p-belief and plausibility measures.*

Session ML3: Friday, May 16, Science Hall 115.

10:30 – 10:50 **Konstantinos Papafilippou** (University of Ghent), *Parametric unification: when projectivity meets uniform post-interpolants.*

11:00 – 11:20 **Alexandru Baltag** (Institute for Logic, Language and Computation), *The topology of surprise.*

11:30 – 11:50 **Sonja Smets** (Institute for Logic, Language and Computation), *Reasoning about quantum information: the probabilistic logic of quantum programs.*

MT. Model Theory

(Organized by Gabe Conant and Nick Ramsey)

Session MT1: Tuesday, May 13, Science Hall 108.

2:00 – 2:20 **Aaron Anderson** (University of Pennsylvania), *Examples of distal metric structures.*

2:30 – 2:50 **Diego Bejarano** (UC Berkeley), *Definability and Scott rank in separable metric structures.*

3:00 – 3:20 **David Meretzky** (University of Notre Dame), *Differential Galois theory with new algebraic constants.*

Session MT2: Wednesday, May 14, Science Hall 108.

2:00 – 2:50 **Michael C. Laskowski** (University of Maryland), *Equivalents of NOTOP.*

3:00 – 3:20 **Christine Eagles** (University of Waterloo), *A uniqueness condition for composition analyses.*

3:30 – 3:50 **Michele Bailetti** (Wesleyan University), *A walk on the wild side.*

Session MT3: Thursday, May 15, Science Hall 108.

2:00 – 2:50 **Lynn Scow** (California State University, San Bernardino), *The modeling property.*

3:00 – 3:20 **Léo Jimenez** (The Ohio State University), *Internality of autonomous systems of differential equations.*

3:30 – 3:50 **Aris Papadopoulos** (University of Maryland), *Mekler's construction and Murphy's law for 2-nilpotent groups.*

PA. Proof Assistants

(Organized by Patricia Johann and Jonathan Weinberger)

Session PA1: Thursday, May 15, Science Hall 107.

12:40 – 1:20 **Egbert Rijke** (Johns Hopkins University), *Mathematical structures from a univalent point of view.*

1:30 – 1:45 **Patricia Johann** (Appalachian State University), *Deep induction for advanced data types.*

1:55 – 2:10 **Talitha Holcombe** (Chapman University), *A common abstract syntax for total functional programming and interactive theorem provers.*

2:20 – 3:00 **Michael Shulman** (University of San Diego), *An observational proof assistant for higher-dimensional mathematics.*

3:10 – 3:25 **Wojciech Nawrocki** (Carnegie Mellon University), *Compiling homotopy type theory with Lean: syntax and interpretation.*

3:30 – 3:45 **Spencer Woolfson** (Carnegie Mellon University), *Compiling homotopy type theory with Lean: the groupoid model of HoTT0.*

3:55 – 4:10 **Aeacus Sheng** (Carnegie Mellon University), *Formally verifying automata for trusted decision procedures.*

Session PA2: Friday, May 16, Science Hall 107.

10:30 – 11:10 **Leonardo de Moura** (Amazon Web Services), *Verified collaboration: low Lean is transforming mathematics, programming, and AI.*

11:20 – 12:00 **Emily Riehl** (Johns Hopkins University), *Prospects for formalizing the theory of weak infinite-dimensional categories.*

12:05 – 12:20 **Peter Jipsen** (Chapman University), *Representability and formalization of relation algebras.*

ST. Set Theory

(Organized by Dima Sinapova and Clinton Conley)

Session ST1: Tuesday, May 13, Science Hall 107.

2:00 – 2:30 **Natasha Dobrinen** (University of Notre Dame), *Ramsey spaces and their ultrafilters.*

2:40 – 3:10 **Anton Bernshteyn** (University of California, Los Angeles), *Borel Local Lemma for graphs of slow growth.*

3:20 – 3:50 **Filippo Calderoni** (Rutgers University), *Idealistic equivalence relations remastered.*

Session ST2: Wednesday, May 14, Science Hall 107.

2:00 – 2:30 **James Cummings** (Carnegie Mellon University), *Linear orderings and singular cardinal combinatorics.*

2:40 – 3:10 **Ruiyuan Chen** (University of Michigan), *Topology versus Borel structure for actions, equivalence relations, and groupoids.*

3:20 – 3:50 **Maxwell Levine** (University of Freiburg) *Namba forcing and singular cardinals.*

Session ST3: Friday, May 16, Science Hall 106.

10:30 – 10:50 **Riley Thornton** (Carnegie Mellon University), *Measurable nibbling and hypergraph limits.*

11:00 – 11:20 **William Adkisson** (University of California Los Angeles), *Tree properties at successors of singulars of many different cofinalities.*

11:30 – 11:50 **Jenna Zomback** (University of Maryland), *Asymptotically spherical groups.*

12:00 – 12:20 **Eyal Kaplan** (University of California, Berkeley), *Failure of GCH on a measurable with the Ultrapower Axiom.*

CONTRIBUTED TALKS

WEDNESDAY, May 14

Session A, 4:30-5:50, Science Hall 106.

4:30 – 4:50 **Jackson West** (New Mexico State University), *Farness logics of Euclidean spaces.*

5:00 – 5:20 **Elijah Gadsby** (Graduate Center CUNY), *Properties of selector proofs.*

5:30 – 5:50 **Arzhang Kamarei** (Kamarei Advisory), *Using paradoxical conditionals to reify and imply a semantic fixed point for Godel's G in first order arithmetic.*

Session B, 4:30-5:50, Science Hall 107.

4:30 – 4:50 **Sapir Ben-Shahar** (University of Wisconsin), *On quasi-reducibility for c.e. sets.*

5:00 – 5:20 **Bjørn Kjos-Hanssen** (University of Hawaiʻi at Mānoa), *The Shannon effect.*

5:30 – 5:50 **Ronald Fuller** (Institute for Logic and the Public Interest), *A new kind of information.*

Session C, 4:30-5:20, Science Hall 108.

4:30 – 4:50 **Morgan Bryant** (University of Maryland), *Merges of smooth classes and their properties.*

5:00 – 5:20 **Connor Lockhart** (University of Maryland), *Model theory of the Farey graph via smooth classes.*

Session D, 4:30-5:50, Science Hall 109.

4:30 – 4:50 **Tan Ozalp** (Notre Dame University), *Initial Tukey structure below a stable ordered-union ultrafilter.*

5:00 – 5:20 **Hongyu Zhu** (University of Wisconsin), *The Borel complexity of the class of models of first-order theories.*

5:30 – 5:50 **Robert S. Lubarsky** (Florida Atlantic University), *On strategies for player II in Σ^0_2 games.*

Abstract of invited tutorial

▸ MARIYA I. SOSKOVA, *Enumeration reducibility and effective mathematics.*
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Relative computability allows us to compare incomputable sets with respect to their algorithmic complexity. The most widely studied way to do this was introduced by Turing [3]: a set of natural numbers X is *Turing reducible* to a set of natural numbers Y if there is an algorithm to determine whether $n \in X$ when given Y as data.

Turing reducibility between sets of natural numbers allows us to gauge the algorithmic content of a mathematical object, such as a real number or a continuous function. For example, Moschovakis [2] observed that every continuous function is computable relative to some fixed Turing oracle, thus understanding relatively computable functions gives a different perspective on the study of continuous functions on the reals. This point of view belongs to *effective mathematics*. However, Turing reducibility is not well suited to handle partial information: suppose that instead of total access to the membership in the oracle, we are only given access to the positive information. Friedberg and Rogers [1] capture this extended model of relative computability: X is *enumeration reducible* to Y if there is an algorithm to enumerate X given any enumeration of Y.

Each reducibility induces a partial order, its degree structure, in which we identify sets that are reducible to each other. There is a way to express Turing reducibility using enumeration reducibility and so we can view the Turing degrees $\mathcal{D}_T$ as a proper substructure of the enumeration degrees $\mathcal{D}_e$. Understanding the algebraic profile of the larger structure results in a better understanding of the smaller one. We need both Turing and enumeration reducibility to study effective mathematics. In this tutorial we will explore various aspects of enumeration reducibility, the induced degree structure, as well as its relationship to the Turing degrees and to effective mathematics.

[1] RICHARD M. FRIEDBERG AND HARTLEY ROGERS, JR., *Reducibility and completeness for sets of integer*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 5 (1959), pp. 117–125.

[2] YIANNIS N. MOSCHOVAKIS, ***Descriptive set theory***, Studies in Logic and the Foundations of Mathematics, North-Holland, 1980.

[3] A. M. TURING, *Systems of Logic Based on Ordinals*, ***Proceedings of the London Mathematical Society***, vol. 45 (1939), no. 3, pp. 161–228.

▸ JULIA WOLF, *An introduction to higher-order stability.*
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For more than half a century, the model-theoretic notion of stability has played a fundamental role in describing tame behaviour, with numerous manifestations across algebra, number theory and combinatorics. In the latter setting, Malliaris and Shelah showed in 2014 that graphs whose edge relation is stable satisfy a particularly strong version of Szemerédi's regularity lemma, a key tool in extremal combinatorics and theoretical computer science since the late 1970s. In this tutorial, I will explain how higher-order generalisations of Szemerédi's regularity lemma, which emerged in the early 2000s in groundbreaking work of Gowers and independently Rödl–Nagle–Skokan–Schacht, have recently pointed the way towards a long-sought generalisation of stability, beyond the case of binary formulas.

We begin by recalling the definitions of stability and NIP, before surveying a cascade of results inspired by Malliaris–Shelah, both in the setting of graphs and finite groups. In particular, we aim to draw out the distinct roles that stability and NIP play in the context of regularity decompositions, as well as the connection between regularity decompositions of finite groups and the model-theoretic "connected component".

We then motivate the ternary notions NIP_2, introduced by Shelah in 2017, and NFOP_2, introduced in joint work with Terry in 2021, and show how their implications in the setting of regularity lemmas for 3-uniform hypergraphs provide compelling evidence for the claim that these two notions do indeed constitute higher-arity analogues of NIP and stability, respectively.

If time permits, we will give an overview of the most recent literature on NFOP_k, including work of Abd-Aldaim–Conant–Terry and Boissonneau-Papadopoulos-Touchard.

Abstracts of invited plenary lectures

▶ SERGEI ARTEMOV, *Consistency of* PA *is a serial property, and it is provable in* PA.
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We revisit the question of whether the consistency of Peano Arithmetic PA can be established in PA. First, we show that Gödel's Second Incompleteness theorem, G2, does not imply a negative answer to this question. Then, we reconfigure Hilbert's epsilon-substitution method of proving consistency and use partial truth definitions to establish PA consistency in PA.

The consistency of a theory means that all its formal derivations are free of contradictions. Derivations are finite syntactic objects, and their Gödel codes are all standard natural numbers. Such numbers can be identified with numerals $\overline{0}, \overline{1}, \overline{2}, \ldots, \overline{n} \ldots$ in the language of PA (we'll drop overlines for better readability). Let $x\!:\!y$ denote the standard primitive recursive proof predicate in PA *"x is code of a proof of a formula having code y"* and $\perp$ stand for the formula $(0=1)$. Consider the series of formulas, **consistency scheme**, $\mathsf{Con}^S_{\mathsf{PA}}$:

$$\neg(0\!:\!\perp), \neg(1\!:\!\perp), \neg(2\!:\!\perp), \ldots, \neg(n\!:\!\perp), \ldots .$$

Since Gödel numbering and proof predicate represent the structure of proofs adequately, $\mathsf{Con}^S_{\mathsf{PA}}$ holds iff PA is consistent.

Although the scheme $\mathsf{Con}^S_{\mathsf{PA}}$ fairly represents PA consistency in the form compatible with the language of PA, $\mathsf{Con}^S_{\mathsf{PA}}$ is not within the scope of the usual provability in PA defined for individual formulas. To address the provability of consistency question, we have either

1. to reformulate consistency as a single PA-formula or
2. to extend the notion of PA-proofs to serial properties in a way formalizable in PA.

For (1), logicians have traditionally considered the **consistency formula** $\mathsf{Con}_{\mathsf{PA}}$:

$$\forall x\ \neg(x\!:\!\perp),$$

in which the informal quantifier *"for all numerals"* hidden in $\mathsf{Con}^S_{\mathsf{PA}}$ is replaced by the formal PA-quantifier "$\forall x$." The problem with the consistency formula is that in PA, $\mathsf{Con}_{\mathsf{PA}}$ is **strictly stronger** than $\mathsf{Con}^S_{\mathsf{PA}}$, hence than PA consistency. Here are some justifications for this claim.

- Conceptual: *"for all numerals"* quantifies over standard numbers $0, 1, 2, \ldots, n \ldots$ and PA consistency is known to hold for each of them. The range of "$\forall x$" includes also nonstandard numbers and, by G2, "$\neg(x\!:\!\perp)$" can fail on some nonstandard x.

- Proof theoretic: $\mathsf{Con_{PA}}$ yields all of $\mathsf{Con}^S_{\mathsf{PA}}$, but $\mathsf{PA} + \mathsf{Con}^S_{\mathsf{PA}}$ does not yield $\mathsf{Con_{PA}}$.
- Model theoretic: in the context of PA provability, we have to consider **all** models of PA, not just the standard model. $\mathsf{Con}^S_{\mathsf{PA}}$ holds in all models whereas $\mathsf{Con_{PA}}$, by G2, fails in some nonstandard models of PA.

Therefore, the unprovability of $\mathsf{Con_{PA}}$ in PA does not answer the question about the provability of PA consistency in PA.

For (2), extending the notion of finite proof to serial properties in PA is imperative. The one formula provability is not directly applicable in metamathematics, where many basic principles are serial properties. We cannot even directly prove in PA its own induction principle or that the product of any two polynomials is a polynomial. For these and similar serial properties F, mathematicians *de facto* accept corresponding contentual selector proofs: PA proves that each instance of F is provable. When selector proofs are defined rigorously, the consistency scheme $\mathsf{Con}^S_{\mathsf{PA}}$ becomes provable in PA.

Other reference points for selector proofs are Hilbert's epsilon substitution method and Brouwer-Heyting-Kolmogorov proofs of universal statements, cf. the detailed discussion in [2].

Definition. Let F be a series of arithmetical formulas $\{F_0, F_1, \dots, F_n, \dots\}$. *A selector proof of F in* PA is a pair of (i) a *selector* which is an operation that given n provides a proof of F_n in PA, and a *verifier* which is a proof in PA that the selector does (i). For this work, selectors are assumed to be explicit primitive recursive operations. A formalized selector proof of F is a pair $\langle s, v\rangle$ with:

(i) an arithmetical term $s(x)$ formalizing the given selector procedure,

(ii) a natural formalization of a given verifier which is a PA-proof v of

$$\forall x[s(x){:}F^{\bullet}(x)].$$

Here $F^{\bullet}(x)$ is a natural primitive recursive term which for each n returns the Gödel number of F_n. This definition of a selector proof in PA naturally extends to selector proofs in other sufficiently strong theories.

Selector proofs are decidable finite objects which subsume the usual proofs. Furthermore, if a serial property $\{F_0, F_1, \dots, F_n, \dots\}$ is provable in PA, then each of F_n is provable in PA. Selector proofs are sound w.r.t. the standard model. This means that selector proofs meet the principal requirement:

whatever is provable is arithmetically true

and, as such, could be endorsed as a correct (and overlooked) extension of the notion of proof from single formulas to serial properties.

Theorem (Provability of Consistency). *The consistency of* PA *in the form of* $\mathsf{Con}^S_{\mathsf{PA}}$ *is selector provable in* PA.

This work refutes the Unprovability of Consistency thesis, UoC, and it removes a principal roadblock of Hilbert's consistency program.

The selector proof of PA consistency was given in [1, 2]. Selector proofs in a general setting have been discussed in [2, 3, 6]. Some technical results useful for the theory of selector proofs were found in [5]. Selector proofs also have been explored by Elijah Gadsby, whose presentation was submitted for this conference. In [4] and in further publications, Detlefsen rejected the G2-based justification of UoC from philosophical finitistic positions. Our work refutes UoC on mathematical grounds and proves PA consistency in PA.

[1] S. Artemov, *The Provability of Consistency*, ***ArXiv preprint***, arXiv:1902.07404, 2019.

[2] S. Artemov, *Serial properties, selector proofs and the provability of consistency*, ***Journal of Logic and Computation***, exae034, 2024.

[3] Y. Cheng, *Current research on Gödel's incompleteness theorems*, ***The Bulletin of Symbolic Logic***, vol. 27 (2021), no. 2, pp. 113–167.

[4] M. Detlefsen, *On interpreting Gödel's second theorem.*, ***Journal of Philosophical Logic***, vol. 8 (1979), pp. 297–313.

[5] A. Ignjatović, *Hilbert's Program and the omega-rule*, ***The Journal of Symbolic Logic***, vol. 59 (1994), no. 1, pp. 322–343.

[6] P. G. Santos, W. Sieg, and R. Kahle, *A new perspective on completeness and finitist consistency*, ***Journal of Logic and Computation***, vol. 34 (2024), no. 6, pp. 1179–1198, exad021.

▸ BENJAMIN CASTLE, *Reconstruction Problems in Geometry via Model Theory.*
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In algebraic geometry, *reconstruction problems* ask for the recovery of an algebraic variety from a restricted set of data. The idea is to determine when a smaller amount of algebraic structure is enough to recover *all* algebraic structure. Geometers typically formulate such problems categorically – however, a natural equivalent formulation is often possible using *definability* in a certain first-order language. In this talk, I will explain the basic set up of *relics* and *full relics* of a first-order structure, and show how – by observations of Zilber – reconstruction problems really amount to determining which relics of a field are full. I will then discuss how powerful tools in stability theory have recently led to a series of classification theorems for relics of fields. Time permitting, I will then give a survey of a new reconstruction theorem in algebraic geometry which was proven using the model-theoretic setup above.

▸ PATRICK LUTZ, *Measure hyperfiniteness and lossless expansion.*
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The structure of the class of countable Borel equivalence relations under Borel reducibility has been a major focus of descriptive set theory over the past few decades. However, many open questions remain, a number of which involve the class of hyperfinite equivalence relations (essentially the simplest nontrivial countable Borel equivalence relations). In order to better understand these questions, Conley and Miller introduced a weakening of Borel reducibility, known as measure reducibility. They then answered the analogues for measure reducibility of several open questions involving hyperfinite equivalence relations. However, they left open at least one such question. Namely, is there a minimal non-hyperfinite equivalence relation under the relation of measure reducibility? Such an object is called a "measure successor of E_0." In ongoing work, Jan Grebík and I have isolated a combinatorial property of group actions on Polish spaces which implies that the associated orbit equivalence relation is a measure successor of E_0. We have also identified several examples of group actions which are plausible candidates for satisfying this condition. The combinatorial property we have identified is a strong form of graph expansion which we call "lossless expansion" after a similar property studied in computer science and combinatorics. I will explain the context for Conley and Miller's question and the combinatorial condition that Grebík and I have isolated and then sketch the main ideas which relate this combinatorial condition to hyperfiniteness.

▸ MARYANTHE MALLIARIS, *New perspectives on model-theoretic complexity.*
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Part of the power of model-theoretic dividing lines comes from having many different equivalent definitions. The talk will discuss several recent theorems in which certain model theoretic dividing lines appear inherently in other fields, and consider what this means on both sides.

▶ ALEJANDRO POVEDA, *Recent developments on the theory of supercompact cardinals.*
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In this presentation we will report on some recent progress on the theory of supercompact cardinals. We will begin presenting various consistency results as well as a conjecture about how the large-cardinal hierarchy of Woodin's Ultimate-L looks like at these latitudes. One of our main theorems is the consistency of every supercompact cardinal being supercompact with inaccessible targets, which answers questions by Bagaria and Magidor. This configuration follows from a new axiom we introduce (Axiom $\mathcal{A}$) which is shown to be consistent with all known large cardinals. We will show that axiom $\mathcal{A}$ yields a Scott-like theorem for Ultimate-L.

On a related note, we shall also report on the effect of Woodin's HOD Hypothesis upon the behavior of supercompact(-like) cardinals in HOD. As a result of our analysis we will answer questions by Cummings–Friedman–Golshani and Cheng–Hamkins–Friedman.

▶ FELIX WEILACHER, *Separating complexity classes of LCL problems on grids.*
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A locally checkable labeling (LCL) problem on a finitely generated group Γ asks one to find a labeling of Cayley graph of Γ satisfying a fixed, finite set of "local" constraints. Such a labeling can equivalently be described as an element of some Γ-subshift of finite type.

How hard is it to solve a particular LCL problem? We consider this question from the point of view of several fields that might be lumped together under the term *definable combinatorics*. The common theme is that we are given a free action of Γ, and want to solve the LCL problem in a "uniform" way on the orbits. For example, in descriptive set theory we might have a continuous action of Γ on a Polish space and wish to solve the problem in a continuous, Borel, etc. way. In computability theory we might have an action of Γ on $\mathbb{N}$ and wish to solve the problem in a computable way.

We study the case $\Gamma = \mathbb{Z}^n$, where we separate various "complexity classes" in definable combinatorics which were not previously known to be distinct. For example, we construct LCL problems on $\mathbb{Z}^n$ which ...

- Have measurable solutions but not necessarily Borel solutions for actions on standard probability spaces.
- Have computable solutions but not necessarily Baire measurable solutions.
- Can be solved as a factor of i.i.d. variables on $\mathbb{Z}^n$, but not as a so-called "finitary" factor.

The first and third items resolve questions of Grebík and Rozhoň. The second and third items are the first examples of such separation for *any* group.

This is joint work with Katalin Berlow, Anton Bernshteyn, and Clark Lyons.

Abstracts of invited talks in the Special Session on
Combinatorics and Logic Theory

▸ SAUGATA BASU AND DEEPAM PATEL, *Cohomological VC-density: bounds and applications.*
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The concept of Vapnik-Chervonenkis (VC) density is pivotal across various mathematical fields, including model theory, discrete geometry, and probability theory. In this paper, we introduce a topological generalization of VC-density. Let Y be a topological space and $\mathcal{X}, \mathcal{Z}^{(0)}, \dots, \mathcal{Z}^{(q-1)}$ be families of subspaces of Y. We define a two parameter family of numbers, $\mathrm{vcd}^{p,q}_{\mathcal{X},\overline{\mathcal{Z}}}$, which we refer to as the degree p, order q, VC-density of the pair

$$(\mathcal{X}, \overline{\mathcal{Z}} = (\mathcal{Z}^{(0)}, \dots, \mathcal{Z}^{(q-1)}).$$

The classical notion of VC-density within this topological framework can be recovered by setting $p = 0, q = 1$. For $p = 0, q > 0$, we recover Shelah's notion of higher-order VC-density for q-dependent families [4]. Our definition introduces a new notion when $p > 0$.

We examine the properties of $\mathrm{vcd}^{p,q}_{\mathcal{X},\overline{\mathcal{Z}}}$ when the families $\mathcal{X}$ and $\mathcal{Z}^{(i)}$ are definable in structures with some underlying topology (for instance, the analytic topology over $\mathbb{C}$, the etale site for schemes over arbitrary algebraically closed fields, or the Euclidean topology for o-minimal structures over $\mathbb{R}$). Our main result establishes that in any model of these theories

$$\mathrm{vcd}^{p,q}_{\mathcal{X},\overline{\mathcal{Z}}} \leq (p+q)\dim X.$$

This result generalizes known VC-density bounds in these structures [3, 2, 1], extending them in multiple ways, as well as providing a uniform proof paradigm applicable to all of them. We give examples to show that our bounds are optimal. Moreover, our bounds on 0/1-patterns actually goes beyond model-theoretic contexts: they apply to arbitrary correspondences of schemes with respect to singular, étale, or ℓ-adic cohomology theories. A particular consequence of our results is the extension of the bound on 0/1-patterns for definable families in affine spaces over arbitrary fields, as initially proven in [3], to general schemes.

We present combinatorial applications of our higher-degree VC-density bounds, deriving topological analogs of well-known results such as the existence of ε-nets and the fractional Helly theorem. We show that with certain restrictions, these results extend to our higher-degree topological setting.

[1] Matthias Aschenbrenner, Alf Dolich, Deirdre Haskell, Dugald Macpherson and Sergei Starchenko, *Vapnik-Chervonenkis density in some theories without the independence property, I*, ***Transactions of the American Mathematical Society***, vol. 368 (2016), no. 8, pp. 5889–5949.

[2] Saugata Basu, *Combinatorial complexity in o-minimal geometry*, ***Proceedings of the London Mathematical Society. Third Series***, vol. 100 (2010), no. 2, pp. 405–428.

[3] L. Rónyai, L. Babai and M. Ganapathy, *On the number of zero-patterns of a sequence of polynomials*, ***Journal of the American Mathemtical Society***, vol. 14 (2001), no. 3, pp. 717–735.

[4] Saharon Shelah, *Strongly dependent theories*, ***Israel Journal of Mathematics***, vol. 204 (2014), no. 1, pp. 1–83.

▸ CHRIS CONIDIS, *The computability of the uniform Krull intersection theorem.*
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The Krull Intersection Theorem (KIT) [1, Theorem 8.9] says that if I is an ideal in a Noetherian integral domain R, then 0 is the only element contained in every power of I. We examine the computability properties of KIT, especially in the uniform context. More precisely, we can show that while the classical proof of uniform Krull implies comprehension for two quantifier predicates, there is also logically simpler argument.

[1] H. Matsumura, ***Commutative Ring Theory***, Cambridge University Press, 2006.

▸ DAVID GONZALEZ, *Ramsey theoretic statements motivated by generic computability.*
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Calvert, Cenzer, and Harizanov introduced several notions of dense computability for countable structures. These tools are used to understand the computable structure theory for weaker, approximate forms of computability. That said, a turn in recent work has revealed that understanding some of these notions is far more structural than initially expected. Efforts in the area have moved in part to studying the substructure relation and its stronger, more elementary counterparts. In particular, looking for well-behaved substructures and elementary substructures of a class of structures in a manner analogous to Ramsey's theorem is key to understanding the generic computability of those structures.

This talk will focus on the structural Ramsey-like theorems that have emerged from this line of research. We will also discuss how and to what extent these results are related to questions regarding generic computability. New results regarding notions of generic categoricity will be emphasized.

This talk is based on joint work with Wesley Calvert, Doug Cenzer, and Valentina Harizanov.

▸ CECELIA HIGGINS, *Complexity of finite Borel asymptotic dimension.*
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A Borel graph is hyperfinite if it can be written as a countable increasing union of Borel graphs with finite components. It is a major open problem in descriptive set theory to determine the complexity of the set of hyperfinite Borel graphs. In a recent paper, Conley, Jackson, Marks, Seward, and Tucker-Drob introduce the notion of Borel asymptotic dimension, a definable version of Gromov's classical notion of asymptotic dimension, which strengthens hyperfiniteness and implies several nice Borel combinatorial properties. We show that the set of locally finite Borel graphs having finite Borel asymptotic dimension is Σ^1_2-complete. This is joint work with Jan Grebík.

▸ SAM MURRAY, *Borel fractional perfect matchings in quasi-transitive amenable graphs.*
Department of Mathematics and Statistics, McGill University, 845 Sherbrooke St W, Montreal.
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In this talk, we show that if a locally finite Borel graph with quasi-transitive amenable components admits a fractional perfect matching, it will admit a Borel fractional perfect matching. In particular, if a countable amenable quasi-transitive graph admits a fractional perfect matching then its Bernoulli graph admits a Borel fractional perfect matching.

► REHANA PATEL, *Countable infinitary theories admitting an invariant measure.*
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The notion of "probabilistic structure" has a long history going back at least to Gaifman in 1964. In a modern framing, these probabilistic structures are closely related to probability measures on the class of structures in a fixed countable language with underlying set $\mathbb{N}$ that are invariant under the logic action. The ergodic such invariant measures, which we call *ergodic structures*, are of particular interest because they admit a satisfaction relation with respect to sentences of the infinitary logic $L_{\omega_1\omega}$. In this talk, we will describe some aspects of a model theory for this satisfaction relation. This is joint work with Nathanael Ackerman and Cameron Freer.

► THEODORE A. SLAMAN, *Cauchy subsequences and the cohesiveness principle.*
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One of the tenets of Reverse Math is that every basic theorem in a typical undergraduate mathematics curriculum is equivalent to one of the "Big Five": Recursive Comprehension, Weak König's Lemma, Arithmetic Comprehension, Arithmetic Transfinite Recursion, or Π^1_1-comprehension. The restriction to typical undergraduate mathematics is necessary since there is a well-documented zoo of combinatorial counter-examples, especially within Ramsey Theory. Counter to the Big-Five-Tenet, one of these combinatorial examples, the Cohesiveness Principle, is formally equivalent to the statement "Every totally bounded sequence of rational numbers has a Cauchy subsequence." This same statement can be used to highlight the role of non-effective methods within a first undergraduate course on Mathematical Logic.

► ATTICUS STONESTROM, *An arithmetic algebraic regularity lemma.*
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In [6], Tao proved an 'algebraic regularity lemma' for families of graphs uniformly definable in finite fields, which improves the conclusions of Szemerédi's regularity lemma in that setting. I will present an 'arithmetic' version of this theorem, ie a version for uniformly definable subsets of uniformly definable groups; this follows in the vein of papers such as [1], [2], [3], [7], and [8] in giving good structure theorems for subsets of groups satisfying additional combinatorial hypotheses.

The precise statement of the theorem is the following: for any $M > 0$, any finite field $\mathbf{F}$, and any definable group $(G, \cdot)$ in $\mathbf{F}$ and definable subset $D \subseteq G$, each of complexity at most M, there is a normal definable subgroup $H \leqslant G$, of index and complexity $O_M(1)$, such that the following holds: for any cosets V, W of H, the bipartite graph $(V, W, xy^{-1} \in D)$ is $O_M(|\mathbf{F}|^{-1/2})$-quasirandom.

When G is the additive group of the field, this result relates in a similar way to Green's theorem from [5] as Tao's result relates to Szemerédi regularity. On the opposite end of the spectrum, when G is the $\mathbf{F}$-points of a simply-connected semisimple algebraic group over $\mathbf{F}$, the result is connected to and largely subsumed by Gowers' work in [4].

This is joint work with Anand Pillay.

[1] Noga Alon, Jacob Fox, and Yufei Zhao, *Efficient arithmetic regularity and removal lemmas for induced bipartite patterns*, ***Discrete Analysis***, vol. 3 (2019).

[2] Gabriel Conant, Anand Pillay, and Caroline Terry, *A group version of stable regularity*, ***Mathematical Proceedings of the Cambridge Philosophical Society***, vol. 168, (2020), no. 2, pp. 405-413.

[3] ——— *Structure and regularity for subsets of groups with finite VC-dimension*, ***Journal of the European Mathematical Society***, vol. 24 (2022), no. 2, pp. 583–621.

[4] Tim Gowers, *Quasirandom groups*, ***Combinatorics, Probability, and Computing***, vol. 17 (2008), no. 3, pp. 363-387.

[5] Ben Green, *A Szemeredi-type regularity lemma in abelian groups, with applications*, ***Geometric and Functional Analysis***, vol. 15 (2005), pp. 340-376.

[6] Terence Tao, *Expanding polynomials over finite fields of large characteristic and a regularity lemma for definable sets*, ***Contributions to Discrete Mathematics***, vol. 10 (2015), no. 1, pp. 22-98.

[7] Caroline Terry and Julia Wolf, *Stable arithmetic regularity in the finite field model*, ***Bulletin of the London Mathematical Society***, vol. 51 (2019), no. 1, pp. 70-88.

[8] ——— *Quantitative structure of stable sets in finite abelian groups*, ***Transactions of the American Mathematical Society***, vol. 373 (2020), pp. 3885-3903.

► ANDY ZUCKER, *Enumerations of Boolean algebras.*
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When attempting to prove big Ramsey results for new structures, exploring recurrent expansions can be a fruitful place to start. We consider the countable atomless Boolean algebra and expansions of it by an enumeration order. While we are far from settling the question of whether or not this structure has finite big Ramsey degrees, we can see by studying its enumerations that it behaves very differently from any Fraisse limit of a relational class with strong amalgamation. For Fraisse limits of strong amalgamation classes, all enumerations are recurrent and in the same recurrence class. For countable atomless Boolean algebras, some enumerations are not recurrent, and among those that are, we describe several different recurrence classes. Joint work with Barbara Csima, Jan Hubicka, and Joey Lakerdas-Gayle.

Abstracts of invited talks in the Special Session on Computability Theory

► ANDREW DELAPO, *Computability and countable second-countable spaces.*
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The study of countable second-countable (CSC) topological spaces within computability theory was initiated by Dorais in 2011 in the context of reverse mathematics. By restricting to CSC spaces, we can analyze the computability-theoretic content of several theorems and constructions in point-set topology. For example, we can consider the fact that every infinite Hausdorff topological space has an infinite discrete subspace. In this talk, we will examine this principle for CSC spaces from multiple perspectives, including computable structure theory, reverse mathematics, and the Weihrauch degrees.

► JACOB FIEDLER, *Extending affine subspaces in higher dimensions.*

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In geometric measure theory, one often seeks to understand how geometric properties or operations influence the size of sets. Suppose $E \subset \mathbb{R}^n$ is the union of a collection of large subsets of k-dimensional planes. If each subset is replaced with the entire k-plane, how does this affect the size of the union? We discuss some recent work on this (classical) problem that uses algorithmic information theory and the point-to-set principle.

► JULIA KNIGHT, KAREN LANGE, AND CHARLES MCCOY CSC, *Computable* Π_2 *Scott sentences.*

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By a result of Scott [4], each countable structure for a countable language L is described up to isomorphism by an $L_{\omega_1\omega}$-sentence, known as a *Scott sentence.* We consider structures that are countably infinite. By a result of A. Miller [2], no such structure has a Σ_2 Scott sentence, so having a Π_2 Scott sentence is as simple as possible. A result of Montalbán [3] yields a nice characterization of the structures (for a fixed countable language) with a Π_2 Scott sentence. *Computable infinitary formulas* involve c.e. disjunctions and conjunctions, so they are in a sense comprehensible. We set out to characterize the structures (for a fixed computable language) with a computable Π_2 Scott sentence. We found some examples and proved some partial results. However, it turns out that there is no nice characterization of the class. The index set is Π^1_1-complete. Rachael Alvir, Barbara Csima, and Harrison-Trainor [1] have also shown this. The two groups worked independently, and the proofs are different.

[1] R. Alvir, B. Csima, and M. Harriston-Trainor, *On the computability of optimal Scott sentences*, manuscript, December 21, 2024.

[2] A. Miller, *The Borel classification of the isomorphism class of a countable model*, ***Notre Dame Journal of Formal Logic***, vol. 24 (1983), no. 1, pp. 22-34.

[3] A. Montalbán, *A robuster Scott rank*, ***Proceedings of the American Mathematical Society***, vol. 143 (2015), no. 12, pp. 5427-5436.

[4] D. Scott, *Logic with denumerably long formulas and finite strings of quantifiers*, ***The Theory of Models*** (John Addison, Leon Henkin, and Alfred Tarski, editors) North-Holland, Amsterdam, 1965, pp. 329-341.

► STEFFEN LEMPP, *Chains and antichains in the Weihrauch lattice.*

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We study the existence and the distribution of "long" chains in the Weihrauch degrees, mostly focusing on chains of uncountable cofinality. We characterize when such chains have an upper bound and prove that there are no cofinal chains (of any order type) in the Weihrauch degrees. Furthermore, we show that the existence of coinitial sequences of non-zero degrees is equivalent to CH. Finally, we explore the extendibility of antichains, providing some necessary conditions for maximality.

This is joint work with Marcone and Valenti.

▶ ANG LI, *Countable Ordered Groups and Weihrauch Reducibility*.

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This paper continues to study the connection between reverse mathematics and Weihrauch reducibility. In particular, we study the problems formed from Maltsev's theorem [1] on the order types of countable ordered groups. Solomon [2] showed that the theorem is equivalent to $\Pi^1_1\text{-}\mathsf{CA}_0$, the strongest of the big five subsystems of second order arithmetic. We show that the strength of the theorem comes from having a dense linear order without endpoints in its order type. Then, we show that for the related Weihrauch problem to be strong enough to be equivalent to $\widehat{\mathsf{WF}}$ (the analog problem of $\Pi^1_1\text{-}\mathsf{CA}_0$), an order-preserving function is necessary in the output. Without the order-preserving function, the problems are very much to the side compared to analog problems of the big five.

[1] Anatoly Mal'tsev, *On ordered groups*, ***Izvestiya Akademii Nauk SSSR. Seriya Matematicheskaya***, vol. 13 (1949), no. 6, pp. 473–482.

[2] Reed Solomon, $\Pi^1_1\text{-}\mathsf{CA}_0$ *and Order Types of Countable Ordered Groups*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 1, pp. 192–206.

▶ ALEXANDER MELNIKOV, *The space of continuous functions: a playground for logic*.

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In my talk, I will present a sequence of results concerning computability-theoretic and descriptive properties of the space $C([0,1];\mathbb{R})$ of continuous functions on $[0,1]$, as well as of spaces $C(K;\mathbb{R})$, where K is compact Polish. I will discuss results that are local (i.e., describe interesting classes of functions) as well as global (i.e., describe the space itself). The results are joint with many co-authors and span over 5+ years of work. They include the characterisation of $C([0,1];\mathbb{R})$ among all separable Banach spaces, the primitive recursive universality of $C([0,1];\mathbb{R})$, an unexpected characterisation of regular (automatic) real functions, and several very recent results related to the effective content of Banach-Stone Duality between $C(K;\mathbb{R})$ and K.

▶ REED SOLOMON, *Strong indivisibility for graphs*.

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A graph is strongly indivisible if whenever it is partitioned into two pieces, one of the pieces is isomorphic to the original graph. Peter Cameron showed there are exactly three strongly indivisible graphs: the countable complete graph, the countable completely disconnected graph, and the random graph. In this talk, I will discuss joint work with Damir Dzhafarov and Andrea Volpi on various effective versions of this classification theorem and connections to reverse mathematics.

▶ KESHAV SRINIVASAN, *Effective ultrapowers of algebraic extensions of* $\mathbb{Q}$.

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We consider a computability-theoretic ultrapower construction for structures. We start with a computable structure, and consider its countable ultrapower over a cohesive set of natural numbers. A cohesive set is an infinite set of natural numbers that is indecomposable with respect to computably enumerable sets. It plays the role of an ultrafilter, and the elements of a cohesive power are the equivalence classes of certain partial computable functions. Thus, unlike many classical ultrapowers, a cohesive power is a countable structure. We focus on the cohesive powers of fields, specifically algebraic extensions of $\mathbb{Q}$. We analyze the algebraic properties of these cohesive powers, and determine just how far their first-order theory is from the original field, utilizing the latest results of number theorists working on generalizations of Hilbert's Tenth Problem to $\mathbb{Q}$ and related fields.

▸ JAVA DARLEEN VILLANO, *Computable categoricity relative to a generic degree*.
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A computable structure $\mathcal{A}$ is said to be computably categorical relative to a degree $\mathbf{d}$ if and only if for all $\mathbf{d}$-computable copies $\mathcal{B}$ of $\mathcal{A}$, there is a $\mathbf{d}$-computable isomorphism $f : \mathcal{A} \to \mathcal{B}$. This relativization of categoricity behaves chaotically in the c.e. degrees, and so a natural inquiry which arises is to investigate how it behaves in non-c.e. degrees. In this talk, we discuss how we can build a computable graph which is not computably categorical but is computably categorical relative to a 1-generic degree $\mathbf{d}$, and how this result is optimal.

Abstracts of invited talks in the Special Session on Modal Logic

▸ ANDREW BACON, *What is the logic of logical necessity?*
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Consider an interpreted modal propositional language in which $\Box$ expresses logical necessity, and let Δ be the set of logical truths in this language; i.e. the logic of logical necessity. What can we say about Δ? Since every element of Δ is a logical truth, every element of Δ should be true. Morever, since $\Box A$ expresses logical necessity, a sentence of the form $\Box A$ should be true simpliciter if and only if $A \in \Delta$.

This naturally leads one to the study of models M such that $M \models \Delta$ and $M \models \Box A$ if and only if $A \in \Delta$, and of logics for which such models exist. I distinguish several special types of logics with this property, and will present some recent joint work with Kit Fine on their existence and their properties.

▸ ALEXANDRU BALTAG, *The topology of surprise*.
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I present a topological modal logic, motivated by a famous epistemic puzzle: the Surprise Exam Paradox. It is an epistemic logic, with modalities for 'knowledge' (modeled as the *universal modality*), 'knowability' (represented by the topological *interior operator*), and 'surprise' (i.e. unknowability of the actual world). The last notion has both a *non-self-referential* reading (modeled by Cantor's *derivative*: the set of limit points of a given set) and a *self-referential* one (modeled by Cantor's *perfect core* of a given set: its largest subset without isolated points). I present a complete axiomatization of this logic, showing that it is decidable and PSPACE-complete, and I apply it to the analysis of the Surprise Exam Paradox (in both its non-self-referential and its self-referential versions).

The results are based on joint work with N. Bezhanishvili and D. Fernandez Duque [1], and the same method was applied in [2] to prove completeness and decidability of full topological mu-calculus (based on Cantor's derivative, interior and universal modality).

[1] Baltag, A., N. Bezhanishvili, and D. Fernandez-Duque, *The topology of surprise*, ***19th International Conference of Knowledge Representation and Reasoning*** (Haifa, Israel), (Gabriele Kern-Isberner, Gerhard Lakemeyer, and Thomas Meyer, editors), International Joint Conference on Artificial Intelligence Organization, 2022, pp. 33–42.

[2] ——— *The topological mu-calculus: completeness and decidability*, ***Journal of the Association for Computing Machinery***, vol. 70 (2023), no. 5, pp. 1–38.

▶ AHMEE CHRISTENSEN, *First-order Fischer Servi logic.*
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The Fischer Servi logic FS is a particularly natural intuitionistic analogue of the minimal classical normal modal logic K. We prove the completeness of a first-order analogue of FS with respect to its expected birelational semantics by introducing a novel model construction, the trace model, and proving a truth lemma. Finally, we consider a number of extensions of the logic for which completeness results can also be obtained with trace model constructions. Among these extensions are first-order intuitionistic analogues of popular normal modal logics like KD and $\mathsf{S4}$.

▶ JOEL LUCERO-BRYAN, *On modal logics arising from the Čech-Stone compactification of ordinals.*
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Topological semantics of modal logic interprets modal diamond as topological closure, and hence modal box as topological interior. Its roots can be traced back to the late 1930s and early 1940s in the work of Stone, Tarski, and others. As a consequence of Kuratowski's closure axioms holding in each topological space, Lewis's well-known modal system $\mathsf{S4}$ is the basic modal logic under these semantics. The celebrated McKinsey-Tarski theorem states that $\mathsf{S4}$ is the modal logic of any separable dense-in-itself metrizable space [3]. Utilizing the Axiom of Choice, Rasiowa and Sikorski showed that the McKinsey-Tarski theorem remains true without the separability assumption [4]. But further dropping the dense-in-itself assumption introduces modal logics other than $\mathsf{S4}$. A complete description of such logics was given in [1].

Much less is known beyond metrizable spaces. In [2] it was shown that $\mathsf{S4.1.2}$ is the modal logic of the Čech-Stone compactification $\beta(\omega)$ of the ordinal ω equipped with the interval topology. In 2020, Valentin Shehtman posed two related problems:

- **P1**: For each nonzero $n \in \omega$, axiomatize the modal logic $\mathsf{L_n}$ of the Čech-Stone compactification $\beta(\omega^n)$ of the ordinal ω^n equipped with the interval topology.
- **P2**: Describe the modal logics arising from the Čech-Stone compactification of an arbitrary ordinal under the interval topology.

This talk provides partial solutions to both **P1** and **P2**. Specifically, we use the Continuum Hypothesis to provide a finite axiomatization of the modal logic L_2 of $\beta(\omega^2)$. This solves **P1** for the case $n = 2$ under the Continuum Hypothesis. We also utilize the Cantor normal form of an ordinal to demonstrate the key role of the the logics L_n in solving **P2**.

This is joint work with Guram Bezhanishvili, Nick Bezhanishvili, and Jan van Mill.

[1] G. Bezhanishvili, D. Gabelaia, J. Lucero-Bryan, *Modal logics of metric spaces*, ***Review of Symbolic Logic***, vol. 8 (2015), no. 1, pp. 178–191.

[2] G. Bezhanishvili and J. Harding, *The modal logic of* $\beta(\mathbb{N})$, ***Archive for Mathematical Logic***, vol. 48 (2009), no. 3-4, pp. 231–242.

[3] J. C. C. McKinsey and A. Tarski, *The algebra of topology*, ***Annals of Mathematics***, vol. 45 (1944), no. 2, pp. 141–191.

[4] H. Rasiowa and R. Sikorski, ***The mathematics of metamathematics***, Monografie Matematyczne, Tom 41, Państwowe Wydawnictwo Naukowe, 1963.

▶ JOSEPH MCDONALD, *Monadic ortholattices: completions and duality.*
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An *ortholattice* is a bounded lattice equipped with an order-inverting involutive complementation. A *monadic ortholattice* is an ortholattice equipped with a closure operator, known as a *quantifier*, whose closed elements form a sub-ortholattice. Monadic ortholattices generalize monadic Boolean algebras – the algebraic model of the classical predicate calculus in a single variable. Janowitz [5] first considered quantifiers on orthomodular lattices, and Harding [3] studied them, as well as cylindric ortholattices, for their connections to von Neumann algebras, in particular, to subfactors.

We show that the variety of monadic ortholattices is closed under MacNeille and canonical completions. In each case, the completion of A is obtained by forming an associated dual space X_A that is a monadic orthoframe. This is a set equipped with an orthogonality relation and an additional binary relation satisfying certain conditions. For the MacNeille completion, X_A is formed from the non-zero elements of A, and for the canonical completion, X_A is formed from the proper filters of A. The corresponding completion of A is then obtained as the complete ortholattice of bi-orthogonally closed subsets of X_A with an additional operation defined through the binary relation on X_A.

With the introduction of a suitable topology on a monadic orthoframe, we obtain a dual equivalence between the category of monadic ortholattices and homomorphisms and the category of monadic orthospaces and certain continuous frame morphisms.

This talk is based on joint work with John Harding and Miguel Peinado [4].

[1] Katalin Bimbó, *Functorial duality for ortholattices and De Morgan lattices*, ***Logica Universalis***, vol. 1 (2007), no. 2, pp. 311–333.

[2] Robert Goldblatt, *The Stone space of an ortholattice*, ***Bulletin of the London Mathematical Society***, vol. 7 (1975), no. 1, pp. 45–48.

[3] John Harding, *Quantum monadic algebras*, ***Journal of Physics A: Mathematical and Theoretical***, vol. 55 (2023). no. 39

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▶ ERIC PACUIT AND LEO YANG, *Common p-belief and plausibility measures.*
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Aumann's famous Agreeing to Disagree Theorem [1] states that if a group of agents share a common prior, update their beliefs by Bayesian conditioning based on private information, and have common knowledge of their posterior beliefs regarding some event, these posteriors must be identical. There is an elegant generalization of this theorem by Monderer and Samet [4], later refined by Neeman [5]: if a group of agents share a common prior, update their beliefs using Bayesian conditioning on private information, and have **common p-belief** of their posteriors, these posteriors must be close (i.e., they cannot differ by more than $1 - p$). Here, common p-belief generalizes the concept of common knowledge to probabilistic beliefs: agents commonly p-believe an event E if everyone believes E to at least degree p, everyone believes to at least degree p that everyone believes E to at least degree p, and so on.

This paper further extends the Monderer-Samet-Neeman Agreement Theorem from classical probability measures to plausibility measures—a very general framework introduced by Halpern [2, 3] that unifies many formal models of belief. To facilitate this extension, we provide a new proof of the Monderer-Samet-Neeman theorem. Building upon both the original proof and our new proof, we offer two different generalizations of the theorem to plausibility-based structures.

Our generalized version of the Monderer-Samet-Neeman theorem, based on conditional plausibility measures, provides a unified framework that identifies the key properties of belief essential for demonstrating that common belief in the posterior probabilities of an event implies that these probabilities must be close. This result deepens our theoretical understanding of agreement theorems. We apply these generalized results to various non-classical belief models, including conditional probability structures and lexicographic probability structures. Furthermore, we demonstrate that whenever the conditions of our generalized theorems are not met, the Monderer-Samet-Neeman Agreement Theorem does not hold. Consequently, our findings suggest that we have identified the minimal conditions required for a belief model to satisfy the Monderer-Samet-Neeman Agreement Theorem.

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▶ FEDOR PAKHOMOV AND JUAN P. AGUILERA, *The logic of correct models.*
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The present talk is based on the paper [1]. The provability interpretation of modal logic has been classically considered. R. Solovay [2] proved that if we interpret $\Box\varphi$ as "φ is provable in PA", then Gödel-Löb logic GL precisely axiomatizes all universally provable facts in PA about such an interpretation. Although less widely known, this paper by Solovay also studied an analogous question for the set-theoretic interpretation of $\Box\varphi$ as "φ is true in all models V_κ, where κ is a strongly inaccessible cardinal".

In the present talk, I will discuss a set-theoretic interpretation of a polymodal language with modalities $\Box_n$ indexed by natural numbers, where each $\Box_n\varphi$ is interpreted as "φ holds in all Σ_{n+1}-correct transitive sets". For ZFC as the base theory, such an interpretation is correct for the polymodal provability logic GLP introduced by G.

Japaridze [3] to characterize the logic of graded families of provability predicates in formal arithmetic (although the question of completeness remains open). We focus our attention on the case of the constructible universe, i.e., the theory $\mathsf{ZFC} + L = V$ as the base theory. In this case, we show that the logic of this set-theoretic interpretation is the logic $\mathsf{GLP}.3$, which is the logic GLP extended by the axiom of linearity 3 for each $\Box_n$.

The key result about the logic $\mathsf{GLP}.3$ that allows us to prove this completeness is that $\mathsf{GLP}.3$ is the maximal normal extension of GLP that does not prove $\Box_n\bot$ for any n. It should also be noted that $\mathsf{GLP}.3$ coincides with the logic of closed substitutions of GLP. Since the axiom of constructivity trivially implies the axioms of linearity, the maximality of $\mathsf{GLP}.3$ ensures that it is precisely the logic of the correct model interpretation over $\mathsf{ZFC} + L = V$. Thus, the technical complexities of the result lie mainly within the domain of modal logic, for which we develop a technique based on the use of the sublogic $\mathsf{J}.3$ of $\mathsf{GLP}.3$, which is Kripke-complete, unlike the Kripke-incomplete $\mathsf{GLP}.3$.

[1] J. P. Aguilera and F. Pakhomov, *The logic of correct models*, ***Journal of Mathematical Logic***, vol. 25 (2025), no. 1, pp. 1–24. (accepted for publication).

[2] R. M. Solovay, *Provability interpretations of modal logic*, ***Israel Journal of Mathematics***, vol. 28 (1976), no. 1, pp. 33–71.

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▶ KONSTANTINOS PAPAFILIPPOU, *Parametric Unification: When Projectivity meets Uniform Post-Interpolants*.
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A notion of relativised admissibility has proven itself useful in the characterisation of the provability logic of Heyting Arithmetic [1]. A generalisation of this notion arises from studying a parametric form of Unification. In this context, we found a surprising interplay between projectivity (in the sense introduced by S. Ghilardi [4]) and the uniform post-interpolant for the classical and intuitionistic propositional logic. In particular, we explore whether a projective substitution of a formula is equivalent to its uniform post-interpolant, assuming the substitution leaves the variables of the interpolant unchanged. We show that in classical logic, this holds for all formulas. Although such a nice property is missing in intuitionistic logic, we provide a Kripke semantic characterisation for propositions with this property.

A study on a distinct type of parametric unification was performed recently by R. Nicolau Almeida and S. Ghilardi [3] found similar results. All of this suggests a deeper connection between Uniform Interpolation and Unification style notions warranting further study.

[1] Mojtaba Mojtahedi, *On Provability Logic of HA*, arXiv:2206.00445, 2022

[2] Mojtaba Mojtahedi and Konstantinos Papafilippou, *Projectivity meets Uniform Post-Interpolant: Classical and Intuitionistic Logic*, ***Advances in Modal Logic*** vol. 15, (Agata Ciabattoni, David Gabelaia and Igor Sedlár, editors), College Publications, 2024, pp. 549–564.

[3] Rodrigo Nicolau Almeida and Silvio Ghilardi, *Unification With Simple Variable Restrictions and Admissibility of Π_2-Rules*, ***Advances in Modal Logic*** vol. 15, (Agata Ciabattoni, David Gabelaia and Igor Sedlár, editors), College Publications, 2024, pp. 79–100.

[4] Silvio Ghilardi, *Unification in Intuitionistic Logic*, ***The Journal of Symbolic Logic***, vol. 64 (1999), no. 2, pp. 859–880.

▸ VLADISLAV SLIUSAREV, *Criteria of local tabularity of products of modal logics.*
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A logic is *locally tabular*, if each of its finite-variable fragments contains only a finite number of pairwise nonequivalent formulas. It is well known that for unimodal transitive logics, local tabularity is equivalent to finite height [8], [7]. In the non-transitive unimodal, and in the polymodal case, no axiomatic criterion of local tabularity is known.

We study locally tabular *products of modal logics.* For frames $F = (X, R)$ and $G = (Y, S)$, the *product frame* $F \times G$ is the frame $(X \times Y,\ R^h, R^v)$, where

$$R^h = \{((a,\, b),\, (a',\, b)) \mid b \in Y,\, aRa'\}; \quad R^v = \{((a,\, b),\, (a,\, b')) \mid a \in X,\, bSb'\}.$$

The *product* $L_1 \times L_2$ *of logics* L_1, L_2 is the bimodal logic of the class of products

$$\{F \times G \mid F \models L_1,\, G \models L_2\}.$$

Local tabularity is established for some families of products. The products with the logic of equivalence relations $S5$ provide a valuable example: N. Bezhanishvili [5] showed that every extension of $S5 \times S5$ is locally tabular, while $S5 \times S5$ itself lacks the local tabularity [6]. V. Shehtman [9] identified a family of locally tabular modal products. For close systems, *intuitionistic modal logics*, G. Bezhanishvili et al. described several locally tabular families in [1], [3], [2], and a recent manuscript [4].

In the product $L_1 \times L_2$ of two Kripke complete consistent logics, local tabularity of L_1 and L_2 is necessary for local tabularity of $L_1 \times L_2$. However, the product of two locally tabular logics can be not locally tabular. The simplest example is the logic $S5 \times S5$ [6] . We provide extra semantic and axiomatic conditions which give criteria of local tabularity of the product of two locally tabular logics: bounded cluster property of one of the factors; a condition we call *product reducible path property*; finiteness of the one-variable fragment of the product.

We discuss several applications of the criteria. We describe new families of locally tabular product logics. We show that the local tabularity is not sufficient for the product finite model property. We give an axiomatic criterion of local tabularity for all extensions of the logic $S4.1BD_2 \times S5$ and introduce a new prelocally tabular logic above $S4 \times S5$.

This talk is based on a joint work with Ilya Shapirovsky, which can be accessed as a preprint at http://arxiv.org/abs/2404.01670.

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[4] Guram Bezhanishvili and Chase Meadors, *Local finiteness in varieties of MS4-algebras*, ***The Journal of Symbolic Logic***, Published online, (2024).

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[8] K. Segerberg, *An essay in classical modal logic*, ***Filosofiska studier utgivna av Filosofiska institutionen vid Stockholms universitet***, vol. 13 (1971), no. 1.

[9] V. B. Shehtman, *Squares of modal logics with additional connectives*, ***Uspekhi Matematicheskikh Nauk***, vol. 67 (2012), pp. 129–186.

[10] ——— *Segerberg squares of modal logics and theories of relation algebras*, ***Larisa Maksimova on Implication, Interpolation, and Definability*** (Sergei Odintsov), Springer International Publishing, Switzerland, 2018, pp. 245–296.

▶ SONJA SMETS, *Reasoning about quantum information: the probabilistic logic of quantum programs.*
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To reason about quantum information, I introduce a probabilistic extension of the Logic of Quantum Programs (LQP) in [2]. This framework is capable of expressing important features of quantum measurements and unitary evolutions of multipartite states. The logic [1, 3] includes dynamic modalities $[\pi]$ (for quantum programs π), spacial modalities (to talk about subsystems and local information) and a probabilistic modality to capture the probability that a given test (of a quantum-testable property) will succeed. The probabilistic ingredient greatly enhances the expressivity of the logic, allowing us to use it for the specification and verification of probabilistic quantum protocols. In this presentation, I will explain the main ingredients of the logical system, illustrate its application in quantum information theory and show that the logic, when interpreted on finite-dimensional Hilbert spaces, is decidable. The results in this presentation are based on joint work with A. Baltag et al. in [1] and the decidability result itself extends on an idea employed in the proof of [4].

[1] A. Baltag, J. Bergfeld, K. Kishida, J. Sack, S. Smets, and S. Zhong, *PLQP & Company: Decidable Logics for Quantum Algorithms*, ***International Journal of Theoretical Physics***, vol. 53 (2014), no. 10, pp. 3628–3647.

[2] A. Baltag and S. Smets, *LQP: The Dynamic Logic of Quantum Information*, Special Issue on Quantum Programming Languages, ***Mathematical Structures in Computer Science***, vol. 16 (2006), no. 3, pp. 491–525.

[3] ——— *Reasoning About Quantum Information: An Overview of Quantum Dynamic Logic*, ***Applied Sciences***, Special Issue on Quantum Logics, vol. 12 (2022), no. 9, paper 4458.

[4] M. Dunn, T.J. Hagge, L.S. Moss, and Z. Wang, *Quantum Logic as Motivated by Quantum Computing*, ***The Journal of Symbolic Logic***, vol. 70 (2005), no. 2, pp. 353–359.

Abstracts of invited talks in the Special Session on Model Theory

▶ AARON ANDERSON, *Examples of distal metric structures.*
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We identify several examples of distal metric structures and examine several consequences of distality, such as the existence of distal cell decompositions, in each. These results include joint work with Itaï Ben Yaacov and with Diego Bejarano.

One class of examples starts with finding a metric structure whose automorphism group is the group of increasing homeomorphisms of the unit interval. We will discuss some properties of this structure and extrapolate to other models of its theory, which we call "dual linear continua."

Another source of examples includes real closed metric valued fields. These give rise to a notion of ordered metric structure, providing a viewpoint to study o-minimality in continuous logic.

▸ MICHELE BAILETTI, *A walk on the wild side*.
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The tame sides of dividing lines are often defined by the absence of local combinatorial properties. Using the concept of patterns of consistency and inconsistency, we describe a general framework for talking about dividing lines. Taking this idea to its limit, we introduce and study various notions of maximal complexity for first-order theories.

▸ DIEGO BEJARANO, *Definability and Scott rank in separable metric structures*.
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In [2], Ben Yaacov et. al. extended the basic ideas of Scott analysis to metric structures in infinitary continuous logic. These include back-and-forth relations, Scott sentences, and the Lopez-Escobar theorem to name a few.

In this talk, I will talk my work connecting the ideas of Scott analysis to the definability of automorphism orbits and a notion of isolation for types within separable metric structures.

Our results are a continuous analogue of the robuster Scott rank developed by Montalbán in [3] for countable structures in discrete infinitary logic. However, there are some differences arising from the subtleties behind the notion of definability in continuous logic.

[1] Diego Bejarano, *Definability and Scott rank in separable metric structures*, arXiv.2411.01017.

[2] Itaï Ben Yaacov, Michal Doucha, Andre Nies, and Todor Tsankov, *Metric Scott analysis*, ***Advances in Mathematics,*** vol. 318 (2017), pp.46–87.

[3] Antonio Montalbán, *A robuster Scott rank*, ***Proceedings of the American Mathematical Society***, vol.143 (2015), no.12, pp.5427–5436.

▸ BLAISE BOISSONNEAU, ARIS PAPADOPOULOS AND PIERRE TOUCHARD, *Mekler's construction and Murphy's law for 2-nilpotent groups*.
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Given a model-theoretic dividing line admitting a combinatorial definition (think "independence property"), it is relatively easy to construct a purely combinatorial structure (by which I, of course, mean a graph) which lies precisely on one side of the divide. However, constructing a purely algebraic structure (say a group) with the same model-theoretic behaviour is, a priori, a somewhat more mysterious task.

Mekler's construction comes to the rescue, allowing us to build (2-nilpotent) groups from (nice) graphs in a way that preserves the combinatorial complexity of the graph we started with. This is, of course, ancient news.

In joint work with Boissonneau and Touchard we prove that the fact that Mekler's construction preserves dividing lines such as stability and NIP is no accident. These are special cases of a more general theorem: Mekler's construction preserves all dividing lines admitting a definition through an "indiscernible collapse".

The goal of my talk is to recall Mekler's construction, gloss over a key relative quantifier-elimination result, and discuss how when things *can* go wrong, they *do*.

▶ CHRISTINE EAGLES, *A uniqueness condition for composition analyses.*
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It is well known that in stable theories, we can understand finite dimensional types in terms of minimal types. We will talk about one one such method which we call a composition analysis. We explore a uniqueness condition for a set of minimal types we associate to a type through the composition analysis. This is based on current work in progress.

▶ LÉO JIMENEZ, *Internality of autonomous systems of differential equations.*
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When solving a differential equation, one sometimes finds that solutions can be expressed using a finite number of fixed, particular solutions, and some complex numbers. As an example, the set of solutions of a linear differential equation is a finite-dimensional complex vector space. This is an incarnation of the model-theoretic phenomenon of internality to the constants in a differentially closed field of characteristic zero. In this talk, I will discuss some recent progress, joint with Christine Eagles, on finding methods to determine whether or not the solution set of a differential equation is internal. A corollary of our method also gives a criteria for solutions to be orthogonal to the constants, and in particular not Liouvillian. I will show a concrete application to Lotka-Volterra systems.

▶ MICHAEL C. LASKOWSKI AND DANIELLE S. ULRICH, *Equivalents of NOTOP.*
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A countable theory T is *classifiable* if it is superstable and has both NDOP and NOTOP. For such theories, every model is determined up to isomorphism by an independent tree of countable, elementary substructures. Historically, superstable theories with NOTOP were only studied under the assumption of NDOP, but we prove that countable, superstable theories with NOTOP are extremely well behaved. In particular, models of such theories determined by an independent tree of countable, elementary substructures up to back and forth equivalence.

Many equivalents of NOTOP are given, and we prove that for countable, superstable theories, NOTOP implies PMOP, which was previously only known for theories with NDOP.

▶ DAVID MERETZKY, *Differential Galois theory with new algebraic constants.*
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A well-known counterexample due to Seidenberg shows that Picard-Vessiot extensions need not exist for arbitrary ordinary homogeneous linear differential equations over arbitrary differential fields. A positive result from Kolchin's school at around the same time (1955-56) shows that inside of a differential closure one can always find a fundamental system of solutions to such an equation over the base field which introduces a Galois extension's worth of new constants. I will give a model theoretic account of the structure of differential field extensions generated by fundamental systems of solutions which introduce a finite (algebraic) extension's worth of new constants, detailing the relationship to the Galois groupoid arising from internality data, and describing a partial Galois correspondence.

► LYNN SCOW, *The modeling property*.
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The term *the modeling property* was coined in [1] to pull together different instances of this phenomenon in the literature and thus create a general object of study. This talk will give an overview of how this study has developed in the author's own work and indicate some new directions.

[1] L. Scow, ***Characterization theorems by generalized indiscernibles***, Ph.D. thesis, ProQuest LLC, 2010.

Abstracts of invited talks in the Special Session on Proof Assistants

► MARIO CARNEIRO, NIKOLAI KUDASOV, EMILY RIEHL, DOMINIC VERITY, AND JONATHAN WEINBERGER, *Prospects for formalizing the theory of weak infinite-dimensional categories*.
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A peculiarity of the ∞-categories literature is that proofs are often written without reference to a concrete definition of the concept of an ∞-category, a practice that creates an impediment to formalization. We describe three broad strategies that would make ∞-category theory formalizable, which may be described as "analytic," "axiomatic," and "synthetic." We then highlight two parallel ongoing collaborative efforts to formalize ∞-category theory in two different proof assistants: the "axiomatic" theory in Lean and the "synthetic" theory in Rzk. We show some sample formalized proofs to highlight the advantages and drawbacks of each approach and explain how you could contribute to this effort.

[1] Nikolai Kudasov, Emily Riehl, and Jonathan Weinberger, *Formalizing*

the ∞-categorical Yoneda lemma, ***CPP 2024: Proceedings of the 13th ACM SIGPLAN International Conference on Certified Programs and Proofs***, (2024), pp. 274–290.

[2] Emily Riehl and Dominic Verity, ***Elements of ∞-category theory***, Cambridge Studies in Advanced Mathematics (194), Cambridge University Press, 2022.

[3] Emily Riehl, *Could ∞-category theory be taught to undergraduates*, ***Notices of the American Mathematical Society***, vol. 70 (2023), no. 5, pp. 727–736.

▸ TALITHA HOLCOMBE, *A common abstract syntax for total functional programming and interactive theorem provers.*
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In this work, we introduce a program conversion tool, hs-to-lean, that uses GHC's *ghc-lib-parser* API to translate Haskell programs into Lean code, which is then validated by the Lean compiler. The repo can be found at https://github.com/holcombet/hs-to-lean/tree/main. The result is a successful compilation of a fragment of Haskell into correct and executable Lean code that users can prove theorems about. Our approach is inspired by recent advances in formal verification of Haskell programs in Coq and, following [1], we currently restrict our attention to total Haskell.

Our compiler produces an AST that serves as a common level of abstraction between a fragment of Haskell and various other theorem provers, including Lean, Coq, and Agda. This allows a given Haskell program or fragment to be translated to a selection of proof assistants, making it portable and accessible to a range of verification efforts. The compilation of code fragments is bidirectional, supporting the translation of Haskell code to a target proof assistant and vice versa. This method exposes an interesting level of abstraction that is applicable to all of the languages involved and produces a more maintainable compiler.

These results contribute to the ongoing work in the formalization and verification of mathematics and programming and present a viable approach to unifying the formal systems of different proof assistants.

[1] Antal Spector-Zabusky, Joachin Brietner, Christine Rizkalla, and Stephanie Weirich, *Total Haskell is Reasonable Coq*, ***Proceedings of the 7th ACM SIGPLAN International Conference on Certified Programs and Proofs*** (Los Angeles, CA, USA), Association for Computing Machinery, 2018, pp. 14-27.

▸ PETER JIPSEN, *Representability and formalization of relation algebras.*
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Abstract relation algebras were defined by Alfred Tarski in 1941 to capture the algebraic properties of binary relations. An interesting question is whether a given relation algebra is representable as an algebra of binary relations. Donald Monk proved in 1964 that the theory of representable relation algebras is not finitely based, and Robin Hirsch and Ian Hodkinson in 2001 showed that it is an undecidable problem whether a finite relation algebra is representable. However, Roger Maddux's concept of n-dimensional bases and Steve Comer's one-point extension method can prove (non)representability for various small algebras. Both methods are based on a two-player game for representability, and we revisit implementations of these algorithms and apply them to relation algebras with up to 32 elements. In particular, to decide representability for all relation algebras with 16 elements, the n-dimensional bases implementation was used in 1993 to prove the nonrepresentability for the last two such algebras. Checking these proofs by hand is rather laborious but can now be done with the help of proof assistants.

Lean is an interactive theorem prover that uses a formal language based on dependent type theory to represent mathematics. Its library of definitions and theorems spans many areas of mathematics, including parts of algebra, logic, order theory and category theory. In joint work with Pace Nelson we develop the theory of relation algebras in the Lean proof assistant. Lean is also an efficient functional programming language, hence this is a useful platform for implementing algorithms and checking mathematical results obtained by computer calculations. We report on the current state of our project without assuming any background about Lean.

▸ PATRICIA JOHANN AND EDWARD MOREHOUSE, *Deep induction and advanced data types.*
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Deep induction provides induction rules for deep data types, i.e., data types that are defined over, or mutually recursively with, (other) such data types. Whereas structural induction traverses only the top-level structure of a data type, leaving any data internal to the top-level structure untouched, deep induction inducts over all of the structured data present. Deep induction was originally developed for the generalization of algebraic data types (ADTs) known as nested types in order to define *structural* induction rules for bushes and other so-called truly nested types. In additional to solving this long-standing problem, deep induction also gives genuinely useful induction rules for deep ADTs such as rose trees. Deep induction has more recently been extended to generalized algebraic data types (GADTs), such as exist in Haskell and Agda. This talk will show how to further extend deep induction to even more advanced data types like inductive families and inductive-inductive types. It will also explain how deep induction can lead to simpler proofs in practice.

▸ LEONARDO DE MOURA, *Verified collaboration: low Lean is transforming mathematics, programming, and AI.*
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Imagine a world where mathematicians, programmers, and AI systems can collaborate with complete trust in each other's work. This is the promise of Lean, an open-source project that's transforming how we approach mathematics, software development, and artificial intelligence. Lean provides machine-checkable proofs, eliminating the need for manual verification and allowing humans and AI to build upon each other's work with unprecedented confidence. By addressing the "Trust Bottleneck," Lean opens doors to cross-disciplinary collaboration. In this talk, we'll explore how Lean is impacting these fields. We'll see how it's providing mathematicians with a new way to construct and verify complex proofs, enabling software developers to rigorously verify critical systems, and creating a foundation for more reliable AI for science and mathematics. We'll also discuss the role of the Lean Focused Research Organization (FRO), a non-profit dedicated to advancing Lean and growing its community. The FRO is driving Lean's development as both a proof assistant and an extensible programming language, empowering users to customize its capabilities for diverse applications. Through real-world examples from academia and industry, we'll discover how Lean is paving the way for a more efficient, reliable, and collaborative future in mathematics, software development, and AI.

▶ WOJCIECH NAWROCKI, STEVE AWODEY, MARIO CARNEIRO, SINA HAZRATPOUR, JOSEPH HUA, AND SPENCER WOOLFSON, *Compiling homotopy type theory with Lean: syntax and interpretation.*
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Synthetic theories simplify mathematical developments by providing domain-specific languages and reasoning principles in which certain constructions become more direct. They have also been found to facilitate strong automated reasoning [1]. Homotopy type theory (HoTT) is a synthetic framework for abstract homotopy theory and higher category theory [2]. Although many proof assistants, such as Cubical Agda, support reasoning with synthetic theories, very few of the intended models of these synthetic constructions have been formalized in a proof assistant. This makes it impossible to formally establish results in classical mathematics using synthetic methods. Furthermore, interpretations of complex synthetic constructions can be difficult to compute by hand.

To demonstrate that proof assistants can assist in interpreting synthetic proofs, we are formalizing the model theory of HoTT0, a simplified fragment of HoTT (with a restricted univalence axiom), in Lean. The system HoTT0 can be interpreted in the category of groupoids, a construction known as the *groupoid model* [2]. In this talk, we present ongoing work that implements HoTT0 as an embedded domain-specific language in Lean. Using the prover's extensible syntax and metaprogramming facilities [4], our aim is to provide users with a library of macros that allow unfolding synthetic HoTT0 constructions into elements of the model, and soundly transferring HoTT0 proofs to classical proofs of statements about groupoids. Our code is available at `https://sinhp.github.io/groupoid_model_in_lean4/`.

[1] Trieu H. Trinh, Yuhuai Wu, Quoc V. Le, He He, and Thang Luong, *Solving olympiad geometry without human demonstrations*, ***Nature***, vol. 625 (2024), no. 7995, pp. 476–482.

[2] The Univalent Foundations Program, ***Homotopy Type Theory: Univalent Foundations of Mathematics***, Institute for Advanced Study, `https://homotopytypetheory.org/book`, 2013.

[3] Martin Hofmann and Thomas Streicher, *The groupoid interpretation of type theory*, ***Twenty-five years of constructive type theory (Venice, 1995)***, vol. 36, pp. 83–111, 1998.

[4] Sebastian Ullrich and Leonardo de Moura, *Beyond Notations: Hygienic Macro Expansion for Theorem Proving Languages*, ***Automated Reasoning - 10th International Joint Conference, IJCAR 2020*** (Paris, France, July 1-4, 2020), 2020, pp. 167–182.

▶ EGBERT RIJKE, *Mathematical structures from a univalent point of view.*
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Libraries of formalized mathematics, as well as the communities working on them, are rapidly growing. Such libraries are written in computer programs called proof assistants, which help the user with verifying the correctness of constructions and proofs.

However, most of the modern proof assistants are based on a foundational system called type theory, which is somewhat different from the de facto foundational system of mathematics, first order logic. Type theory allows for new ways of thinking about mathematics, and one of these ways is using an axiom of Vladimir Voevodsky, the univalent axiom. This axiom expands the notion of equality of mathematical objects so that it coincides with isomorphisms. Mathematicians are used to considering isomorphic objects to be the same for practical purposes, and the univalent axiom formalizes this principle. The univalent axiom changes the way we should think of some mathematical concepts, and in this talk, I will explore some of the basic ways in which mathematics is done from a univalent point of view.

▸ MICHAEL SHULMAN, *An observational proof assistant for higher-dimensional mathematics.*
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In *higher-dimensional type theories*, each type has not just ordinary terms but also a structure of higher-dimensional terms, such as an ∞-groupoid or homotopy space (homotopy type theory [6]), a cubical set (parametric type theory [2]), or a semi-simplicial set (displayed type theory [5]). These theories provide new and powerful reasoning tools for formalizing and working with higher structures.

Existing computational higher-dimensional type theories (e.g., [4, 3, 7]) are *interval-based*, meaning the higher structure is detected by mapping out of an "interval type". An alternative is an *observational* theory, in which the higher structure is defined separately type-by-type; e.g., in homotopy type theory a path in a product type is a pair of paths, while a path between functions is a homotopy, and a path between types is an equivalence (the univalence axiom).

In this talk I will introduce a prototype proof assistant called Narya that implements observational higher-dimensional type theories, including higher observational type theory (the observational version of homotopy type theory), and sketch what we know about its metatheory (e.g., [1]). This is joint work with Thorsten Altenkirch and Ambrus Kaposi.

[1] Thorsten Altenkirch, Yorgo Chamoun, Ambrus Kaposi, and Michael Shulman, *Internal parametricity, without an interval*, ***Principles of Programming Languages 2024***, Proceedings of the ACM on Programming Languages, (Michael Hicks, editor), vol 8, 2024, pp. 2340–2369.

[2] Jean-Philippe Bernardy and Guilhem Moulin, *A computational interpretation of parametricity*, ***Proceedings of the 2012 27th Annual IEEE/ACM Symposium on Logic in Computer Science***, IEEE Computer Society, 2012, pp. 135–144.

[3] Evan Cavallo and Robert Harper, *Internal parametricity for cubical type theory*, ***28th EACSL Annual Conference on Computer Science Logic (CSL 2020)*** (Maribel Fernández and Anca Muscholl, editors), Leibniz International Proceedings in Informatics (LIPIcs), vol. 152, Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, Dagstuhl, Germany, 2020, pp. 13:1–13:17.

[4] Cyril Cohen, Thierry Coquand, Simon Huber, and Anders Mörtberg, *Cubical type theory: a constructive interpretation of the univalence axiom*, arXiv:1611.02108 (2016).

[5] Astra Kolomatskaia and Michael Shulman, *Displayed type theory and semi-simplicial types*, arXiv:2311.18781 (2023).

[6] Univalent Foundations Program, ***Homotopy Type Theory: Univalent***

Foundations of Mathematics, `http://homotopytypetheory.org/book/`, 2013.

[7] Andrea Vezzosi, Anders Mörtberg, and Andreas Abel, *Cubical Agda: A dependently typed programming language with univalence and higher inductive types*, ***Proceedings of the ACM an Programming Languages***, vol. 3 (2019), pp. 87:1–87:29.

► AEACUS SHENG, *Formally verifying automata for trusted decision procedures.*
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Automata-theoretic decision procedures date back to Büchi's work, leveraging finite automata to decide sentences of Presburger arithmetic [1][2]. Extensions of the original procedure are now used to check and discover theorems in combinatorics and number theory [4]. However, unlike SAT solvers, which provide verifiable proof certificates, automata-based decision procedures do not generate formal correctness guarantees as they run. As a result, despite their practical success, concerns remain regarding the reliability and trustworthiness of results obtained using automata-based decision procedures.

Towards addressing this issue, we present our ongoing effort to implement and formally verify automata in Lean [3], which is both a proof assistant and a programming language. By formalizing efficiently-executable automata and proving their properties, we want to ensure that automata-theoretic decision procedures are both provably correct and practically useful.

We begin by introducing the mathematical logic contents behind these decision procedures and provide motivating examples on the type of problems they can solve. Then, we present and explain our implementation and verification of automata in Lean. In the end, we give a preliminary demonstration of how to use our work to prove theorems in Lean.

[1] J. Richard Büchi, *Weak second-order arithmetic and finite automata*, ***Zeitschrift für Mathematische Logik und Grundlagen der Mathematik***, vol. 6 (1960), pp. 66–92. Reprinted in S. Mac Lane and D. Siefkes, editors, ***The Collected Works of J. Richard Büchi***, Springer-Verlag, 1990, pp. 398–424.

[2] J. Richard Büchi, *On a decision method in restricted second-order arithmetic*, ***Logic, Methodology and Philosophy of Science*** (Proc. 1960 International Congress), Stanford University Press, 1962, pp. 1–11.

[3] Leonardo de Moura and Sebastian Ullrich, *The Lean 4 Theorem Prover and Programming Language*, ***Automated Deduction – CADE 28: 28th International Conference on Automated Deduction, Virtual Event, July 12–15, 2021, Proceedings*** (Berlin, Heidelberg), Springer-Verlag, 2021, pp. 625–635.

[4] Jeffrey Shallit, ***The Logical Approach to Automatic Sequences: Exploring Combinatorics on Words with Walnut***, London Mathematical Society Lecture Note Series, Cambridge University Press, Cambridge, 2022.

► SPENCER WOOLFSON, STEVE AWODEY, MARIO CARNEIRO, SINA HAZRATPOUR, JOSEPH HUA, AND WOJCIECH NAWROCKI, *Compiling homotopy type theory with Lean: the groupoid model of HoTT0.*
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Homotopy type theory (HoTT) is a powerful system for formal mathematics [1]. Yet, without invoking metamathematics, its theorems are detached from classical mathematics, limiting their applicability. Our collaborative project aims to bridge this gap by developing a formal compiler that automatically translates HoTT proofs into Lean theorems.

We focus on a fragment of HoTT, termed HoTT0, with a semantic interpretation into the category of 1-groupoids. HoTT0 restricts the univalence axiom to only hold on subuniverses of 0-truncated types (i.e. sets). Despite its limiting appearance, this suffices to develop significant portions of univalent, set-level mathematics expressible in HoTT. Building on prior work that modeled type theory in 1-groupoids using categories with families [2], we adapt this approach to natural model semantics [3] based on polynomial functors [4]. The natural model semantics are modular, allowing for implementations of other models of HoTT. This project also lays the groundwork for compiling other type theories in Lean.

In this talk, I plan to discuss the formal system HoTT0, its 1-groupoid model, and show how some syntactic statements can be translated into classical proofs. I will also discuss what is gained by working in HoTT0, as opposed to extensional Martin-Löf type theory. I hope this talk can demonstrate how Lean can be adapted to work in novel ways to fit the needs of different researchers.

[1] The Univalent Foundations Program, ***Homotopy Type Theory: Univalent Foundations of Mathematics***, Institute for Advanced Study, `https://homotopytypetheory.org/book`, 2013.

[2] Martin Hofmann and Thomas Streicher, *The groupoid interpretation of type theory*, ***Twenty-five years of constructive type theory (Venice, 1995)***, vol. 36, pp. 83–111, 1998.

[3] Steve Awodey, *Natural models of homotopy type theory*, ***Mathematical Structures in Computer Science***, vol. 28, no. 2, pp. 241-286, 2018.

[4] Nicola Gambino and Joachim Kock, *Polynomial functors and polynomial monads*, ***Mathematical proceedings of the Cambridge philosophical society***, vol. 154, no. 1, pp. 153–192, Cambridge University Press, 2013.

Abstracts of invited talks in the Special Session on Set Theory

► WILLIAM ADKISSON, *Tree properties at successors of singulars of many different cofinalities.*
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An old problem of Magidor is to obtain the tree property at every regular cardinal greater than $\aleph_1$. This question can also be posed for strengthenings of the tree property. If there is to be a positive answer to this question, we must obtain the tree property at many successors of singular cardinals; in particular, we must obtain the tree property at successors of singular cardinals of many different cofinalities. Motivated by this problem, from many supercompacts we build a model in which the tree property holds at $\aleph_{\omega+\omega+1}$ and $\aleph_{\omega_i+1}$ for all $0 < i < \omega$ simultaneously. This construction can be modified to obtain the strong tree property, a strengthening of the tree property that is closely linked with strongly compact cardinals; it can be modified further to obtain these properties for much longer sequences of desired cofinalities.

► ANTON BERNSHTEYN, *Borel Local Lemma for graphs of slow growth.*
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The Lovász Local Lemma is an important tool in probabilistic combinatorics. Roughly speaking, it shows the existence of a function satisfying certain combinatorial constraints by checking a set of numerical conditions. In addition to its importance in combinatorics, the Local Lemma has recently found applications in many other fields, such as ergodic theory. In this talk, we address the following question: When can we choose the function whose existence is guaranteed by the Local Lemma to be Borel? Csóka, Grabowski, Máthé, Pikhurko, and Tyros proved a Borel version of the Local Lemma under the assumption that a certain auxiliary graph is of subexponential growth. Unfortunately, their proof only works when the range of the desired function is finite. Using a different approach, we extend their result to the case of continuous range as well as to graphs of limited exponential growth. This is joint work with Jing Yu.

▸ FILIPPO CALDERONI, *Idealistic equivalence relations remastered.*
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URL Address: `https://sites.math.rutgers.edu/ fc327/`.

Howard Becker[1] proved that under analytic determinacy there exists an idealistic equivalence relation that is not an orbit equivalence relation. In this talk we discuss a strengthening of Becker's result:

Theorem. *Assume analytic determinacy. Then there exists an idealistic equivalence relation E that is not class-wise Borel embeddable into any orbit equivalence relation.*

This work aims at better understanding the nuances between idealistic and orbit equivalence relations. Along the way we explain how this is related to the long-standing E_1 conjecture initially formulated by Kechris and Louveau [2]. This is joint work with Luca Motto Ros.

[1] Howard Becker. Idealistic equivalence relations, Unpublished notes, 2001.

[2] Alexander S. Kechris and Alain Louveu, *The classification of hypersmooth borel equivalence relations*, ***Journal of the American Mathematical Society***, vol. 10 (1997), no. 1, pp. 215–242.

▸ RUIYUAN CHEN, *Topology versus Borel structure for actions, equivalence relations, and groupoids.*
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The Becker–Kechris theorem characterizes the topological rigidity intrinsic to a Borel action of a Polish group as consisting of precisely the quotient topologies on each orbit. We show that conversely, every Borel equivalence relation (or more generally, standard Borel groupoid) equipped with a "Borel family" of Polish topologies on each class, satisfying suitable axioms, may be represented in terms of a Polish group action.

▸ JAMES CUMMINGS, *Linear orderings and singular cardinal combinatorics.*
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We prove some results about linear orderings with cardinality the successor of a singular cardinal, using ideas from singular cardinal combinatorics.

▸ NATASHA DOBRINEN, *Ramsey spaces and their ultrafilters.*

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Many classes of topological Ramsey spaces give rise to ultrafilters with partition properties. On the flip side, sometimes forcings that produce ultrafilters with partition properties harbor a topological Ramsey space as a dense subset. The added clarity of Ramsey space techniques allow for fine analysis of the properties of such ultrafilters. In this talk, we will discuss special properties of such ultrafilters such as initial Rudin-Keisler and Tukey structures, complete combinatorics, barren extensions, and preservation under Sacks forcing. We will touch on works of Di Prisco, Dobrinen, Hathaway, Mijares, Nieto, Navarro Flores, Özalp, Todorcevic, Trujillo, and Zheng, and discuss ongoing work of the speaker.

▸ EYAL KAPLAN, *Failure of GCH on a measurable with the Ultrapower Axiom*.
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The Ultrapower Axiom (UA) states that any pair of ultrapowers can be compared by internal ultrapowers. The Axiom was extensively studied by Gabriel Goldberg, leading to a series of striking results.

Goldberg asked whether UA is consistent with a measurable cardinal that violates GCH. The main challenge is that UA is not easily preserved under forcing constructions, especially ones that achieve violation of GCH on a measurable from large cardinal assumptions. For example, such forcings might create normal measures which are incomparable in the Mitchell order – a property that negates UA.

In this talk, we will present a recent result that shows that the failure of GCH on the least measurable cardinal can indeed be forced while preserving UA, starting from the minimal canonical inner model carrying a (κ, κ^{++})-extender. This is a joint work with Omer Ben-Neria.

▸ MAXWELL LEVINE, *Namba Forcing and Singular Cardinals*.
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One way to study the properties of the infinite cardinals is to examine the extent to which they can be changed by forcing, and the extent to which this process can be controlled. In 1969 and 1970, Bukovský and Namba independently showed that $\aleph_2$ can be forced to be an ordinal of cofinality $\aleph_0$ without collapsing $\aleph_1$. The forcings they used and their variants are now known as Namba forcing. We will discuss some surprising connections between Namba forcing and the theory of singular cardinals like $\aleph_\omega$.

▸ CHRISTIAN ROSENDAL AND JENNA ZOMBACK, *Asymptotically spherical groups*.
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A length function ℓ on a group G is a function from G to the nonnegative real numbers satisfying the following for all group elements x and y: $\ell(x) = 0$ if and only if $x = 1_G$, $\ell(x^{-1}) = \ell(x)$, and $\ell(xy) \leq \ell(x) + \ell(y)$.

In this talk, we will investigate the asymptotic behavior of compatible length functions on Polish groups, and in particular, the extent to which a sphere of large radius with respect to one length function looks spherical with respect to another. This is joint work with Christian Rosendal.

▸ RILEY THORNTON, *Measurable nibbling and hypergraph limits*.
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The Frankl–Rödl matching theorem says that, in any sparse enough regular hypergraph of large enough degree, there's matching that covers almost all of the vertices. It was one of the first applications of Rödl's (now ubiquitous) nibble method. In this talk, I will prove a measurable version of the Frankl–Rödl theorem using a measurable version of the nibble method and some results about weak containment for hypergraphs.

[1] Riley Thornton, *Limits of sparse hypergraphs*, arXiv.2410.17483, 2024.

Abstracts of contributed talks

▶ SAPIR BEN-SHAHAR, ROD DOWNEY, AND MARIYA SOSKOVA, *On Quasi-reducibility for c.e. sets.*

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Strong reducibilities naturally arise in the combinatorics of reductions in classical mathematics. They are important for at least two reasons: they give insight into exactly how reductions work in practice; and in many cases a strong reducibility may be more appropriate for a specific area. Quasi-reducibility was introduced by Tennenbaum in the 1960s. On c.e. sets it coincides with $*$-reducibility, which was shown to be relevant to existentially closed groups in work of Belegradek [1]. Strong Quasi-reducibility was introduced and studied by Omanadze [2]. I will introduce Quasi-reducibility and strong Quasi-reducibility and discuss some recent results on the structures of the c.e. degrees that arise from these reducibilities. This is joint work with Rod Downey and Mariya Soskova.

[1] Belegradek, O. V., *Algebraically closed groups*, ***Algebra i Logika***, vol. 13 (1974), pp. 239–255, 363.

[2] Omanadze, Roland Sh., *Quasi-degrees of recursively enumerable sets*, ***Computability and models***, Kluwer/Plenum, New York, 2003, pp. 289–319.

▶ MORGAN BRYANT, *Merges of smooth classes and their properties.*

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Given two Fraïssé-like classes, known as *smooth classes*, each with countable generic limits (a generalization of a Fraïssé limit), we ask whether we can merge the two classes into a new smooth class with a generic limit. We then study which model theoretic properties of the generics of the original classes transfer to the generic of the new class, when it exists. In a different direction, we discuss how merges of smooth classes connect to structural Ramsey theory and the Hrushovski property (also known as EPPA). Generalizing work done in [1], we give some examples of merges of smooth classes which have EPPA and the Ramsey property.

[1] David Evans, Jan Hubička, and Jaroslav Nešetřil, *Automorphism groups and Ramsey properties of sparse graphs*, ***Proceedings of the London Mathematical***

Society, vol. 119 (2019), no. 2, pp. 515–546.

▸ RONALD FULLER, *A new kind of information.*
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In his famous essay *On Denoting*, Bertrand Russell called Frege's distinction between *sense* and *reference* "an inextricable tangle." Others have agreed. Frank Ramsey referred to Russell's essay as "that paradigm of philosophy", and according to the Stanford Encyclopedia of Philosophy, Ramsey might just as easily have said "*the* paradigm of philosophy." So experts have viewed Russell's denial of Frege's model of semantics, and the ideas built around this denial, as a definitive perspective, and perhaps *the* definitive perspective, on the entire edifice of philosophy. The consequences have been disastrous.

Russell's entrenched influence on the philosophy of information has left a hole in our understanding of semantics. Schmid and Swenson puzzled over this hole, asking "What does it mean to collect attributes into a relation? ... This is one of the rather important questions that can hardly be answered from the mathematical point of view..." [1]. The absence of a foundational theory of semantics for the structure of information has been a root cause, I argue, of the most difficult challenges facing modern organizations – one of which, data quality, costs organizations \$3 trillion per year [3].

The theory of *sophotaxis* [2] provides an adequate account of semantics for the structure of information and can inform solutions to these challenges. Examples are shown, and a revised definition of semantic information is given.

[1] Schmid, Hans Albrecht, and J. Richard Swenson, *On the Semantics of the Relational Data Model*, ***Proceedings of the 1975 ACM SIGMOD International Conference on Management of Data*** (San Jose, CA), Association for Computing Machinery, 1975, pp. 211–223.

[2] Fuller, Ronald, and Peter Cardon, *Sophotaxis*, ***The Bulletin of Symbolic Logic***, vol. 23 (2016), no. 1, pp. 128–129.

[3] Redman, Thomas C., *Bad Data Costs the U.S. 3 Trillion Per Year*, ***Harvard Business Review***, September 22, 2016.

▸ ELIJAH GADSBY, *Properties of selector proofs.*
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A serial property is a suitably enumerated sequence $\{F_n\}$ of sentences and is called selector provable in PA if there is a PA-recursive function $s(x)$ such that $PA \vdash \forall x(s(x){:}\ulcorner F_x \urcorner)$ where $x{:}y$ is a suitable proof predicate. These notions were introduced by Artemov in his analysis of the formalization of metamathematics, the topic of his plenary talk. In particular, he argues that the consistency of PA is best represented as a serial property and, as such, is selector provable in PA.

This talk will give an overview of the mathematical properties of selector proofs, focusing on the case where the serial property consists of all instances of a single formula. Along the way, it will be seen that iterated selector proofs can be collapsed into single ones, so that natural attempts to extend Artemov's program along these lines cannot succeed. Furthermore, while primitive recursive selectors are more than sufficient in natural cases, serial properties can be constructed that require arbitrarily complex selectors. Finally, a brief survey of some relevant results from the literature will be given. From these, it can be seen that while PA cannot selector prove the consistency of $PA + Con(PA)$, there are infinitely many proper extensions of PA

whose consistency is selector provable in PA.

▸ ARZHANG KAMAREI, *Using paradoxical conditionals to reify and imply a semantic fixed point for Godel's G in first order arithmetic.*
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Computational models of consciousness do not seem to adequately address the potential logical impossibility of non-Tarskian, Cogito-style, self-referential truth predicates (e.g., "I think, therefore I am"). To address this gap, this paper attempts to push the envelope on first order arithmetic (FOA) self-referential logic, targeting a semantic fixed point for Gödel's G. We first assume two semantic conditionals, reifying the semantic logical ontology of G: (a) $\neg\mathrm{Prov}(\ulcorner G\urcorner) \to \mathrm{Prov}(\ulcorner G\urcorner)$ and (b) $\mathrm{Prov}(\ulcorner G\urcorner) \to \neg\mathrm{Prov}(\ulcorner G\urcorner)$. We next show these can only be satisfied by a unique syntactic fixed point, $G = \neg\mathrm{Prov}(\ulcorner G\urcorner)$. Since this fixed point is paradoxical and self-referential, it's syntax and semantics cannot be determined by a consistent system P and G will not characterize $\mathrm{Prov}(x)$ for any such $x \neq G$. This then creates a unique syntactic-semantic tuple or fixed point characterized as $G \equiv \neg\mathrm{Prov}(\ulcorner G\urcorner)$, where: (i) the semantic conditionals only match this syntactic definition, (ii) the syntax also implies the semantic conditions, and (iii) whose syntactic definition matches Godel's G. Because the semantics are not derived from the syntax, we can formalize this equivalence in FOA without violating the Hilbert–Bernays–Löb conditions or conflicting with Löb's Theorem. This creates a constructive version of Gödel's First Incompleteness Theorem in FOA. Given that G is a unique fixed point, we can then take the complement or negation of G, which may imply the existence of "normal" formulas, i.e., $\neg\neg\mathrm{Prov}(\ulcorner G\urcorner)$ implies Prov. If so, this implies constructively that $P \vdash \mathrm{Prov}(\ulcorner x\urcorner) \to x$ for all such $x \neq G$, i.e., $\neg G \to [\mathrm{Prov}(x) \to x]$. This thus bifurcates the universe of formulas into G and $\neg G$, showing that they can have differently determined relationships between truth and proof and which are constructible. This may help open the door to self-referential logics which may be necessary for the digitization of Cogito.

▸ BJØRN KJOS-HANSSEN, *The Shannon effect.*
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In many settings, almost all objects have nearly the same complexity as the hardest objects.

Specifically, Claude Shannon conjectured [3] that almost all Boolean functions have nearly the same circuit complexity as the hardest function. It was proved by Lupanov [2]. Similar statements hold for plain and prefix-free Kolmogorov complexity. The phenomenon was named the Shannon effect by Lupanov in 1970.

For nondeterministic automatic complexity, the effect follows from an upper bound by Hyde from 2013 and an almost sure lower bound by Kjos-Hanssen in 2021.

The nondeterministic automatic complexity also has an "exact" version, in which a single accepting path is not required, merely a single accepted word of a given length. For this notion, a Shannon effect has been proved as well [1].

Whether the effect pertains to deterministic automatic complexity is not known.

[1] Joey Chen, Bjørn Kjos-Hanssen, Ivan Koswara, Linus Richter, and Frank Stephan, *Languages of words of low automatic complexity are hard to compute*, submitted, 2025.

[2] O. B. Lupanov, *The synthesis of contact circuits*, ***Doklady Akademii Nauk SSSR (N.S.)***, 119:23–26, 1958.

[3] Claude E. Shannon, *The synthesis of two-terminal switching circuits*, ***Bell***

System Technical Journal, 28:59–98, 1949.

▸ CONNOR LOCKHART, *Model theory of the Farey graph via smooth classes.*
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We study the model theory of the Farey graph F by realizing it as the generic of a smooth class $(K, \leq)$. By varying the relation $\leq$ we may obtain distinct generics that are either atomic or saturated. This will allow us to demonstrate a quantifier elimination for $Th(F)$. The Farey graph is also the simplest nontrivial curve complex of a surface, where $F = C(\Sigma_{1,1})$. Modifications of this technique to obtain results for the general model theory of the curve complex $C(\Sigma_{g,n})$ will be discussed.

▸ JUAN AGUILERA AND ROBERT S. LUBARSKY, *On strategies for player II in Σ^0_2 games.*
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That certain strategies for Player II in Σ^0_2 games are always winning can be witnessed in L by certain ordinals. There is already a known description of these ordinals. What is less understood is where the strategies themselves appear. What is also not understood is the model theory Player II would use in these strategies. This talk is an introduction to this topic, with the goal of conveying these open questions and some of the ideas around them.

[1] Fred Abramson and Gerald Sacks, *Uncountable Gandy Ordinals*, ***Journal of the London Mathematical Society***, vol. 14 (1976), no. 2, pp. 387–392

[2] Juan Aguilera and Robert Lubarsky, *On winning strategies for F_σ games*, ***The Journal of Symbolic Logic***, to appear.

[3] Richard Gostanian, *The Next Admissible Ordinal*, ***Annals of Mathematical Logic***, vol. 17 (1979), pp. 171–203

[4] Christoph Heinatsch and Michael Möllerfeld, *The determinacy strength of Π^1_2-comprehension*, ***Annals of Pure and Applied Logic***, vol. 161 (2010), pp. 1462–1470

[5] Yiannis Moschovakis, *Descriptive Set Theory*, 2nd edition, American Mathematical Society, 2009

[6] P. Wolfe, *The strict determinateness of certain infinite games*, ***Pacific Journal of Mathematics***, vol. 5 (1955), pp. 841-847

▸ TAN OZALP, *Initial Tukey structure below a stable ordered-union ultrafilter.*
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For arbitrary partial orders $(\mathbb{P}, \leq_{\mathbb{P}})$ and $(\mathbb{Q}, \leq_{\mathbb{Q}})$, $(\mathbb{P}, \leq_{\mathbb{P}})$ is said to be *Tukey reducible to* $(\mathbb{Q}, \leq_{\mathbb{Q}})$, written $(\mathbb{P}, \leq_{\mathbb{P}}) \leq_T (\mathbb{Q}, \leq_{\mathbb{Q}})$, if there is a map $f : \mathbb{Q} \to \mathbb{P}$ which sends cofinal subsets of $\mathbb{Q}$ to cofinal subsets of $\mathbb{P}$. In particular, we can restrict our attention to directed partial orders of the form $(\mathcal{U}, \supseteq)$, where $\mathcal{U}$ is an ultrafilter. In this case, Tukey reducibility generalizes RK-reducibility.

A detailed study of the Tukey order of ultrafilters on countable sets was initiated by Dobrinen and Todorcevic in [2]. Later, Todorcevic proved that Ramsey ultrafilters are Tukey-minimal ([5]). This study continued with Dobrinen and Todorcevic's classification of isomorphism classes of ultrafilters Tukey reducible to a weakly Ramsey ultrafilter ([3]). The rest of this line of work and more detailed information can be found in [1]

Dobrinen and Todorcevic asked the question of classification of ultrafilters Tukey reducible to a stable ordered-union ultrafilter ([2]). The goal of this talk will be to introduce stable ordered-union ultrafilters on FIN, and state the classification of all ultrafilters Tukey reducible to a stable ordered-union $\mathcal{U}$. This is the main result of [4], and answers a question from [2]. I will also talk about the techniques that were used in the proof of this classification, and finish with remarks on a canonization theorem for $\mathrm{FIN}_2^{[\infty]}$ that I am currently finalizing, that will be used to classify all ultrafilters Tukey reducible to the ultrafilter forced by $\mathrm{FIN}_2^{[\infty]}$.

[1] N. Dobrinen, *Topological Ramsey spaces dense in forcings*, ***Structure and Randomness in Computability and Set Theory*** World Scientific Publishing, Hackensack, NJ, 2020, pp. 3–58.

[2] N. Dobrinen and S. Todorcevic, *Tukey types of ultrafilters*, ***Illinois Journal of Mathematics***, vol. 55 (2011), no. 3, pp. 907–951.

[3] ——— *A new class of Ramsey-classification theorems and their application in the Tukey theory of ultrafilters, Part 1*, ***Transactions of the American Mathematical Society***, vol. 366 (2014), no. 3, pp. 1659–1684.

[4] T. Ozalp, *Initial Tukey structure below a stable ordered-union ultrafilter*, arXiv:2410.04326 (2024).

[5] D. Raghavan and S. Todorcevic, *Cofinal types of ultrafilters*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 3, pp. 185–199.

▸ JACKSON WEST, *Farness logics of Euclidean spaces.*
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In any metric space (X, d), we can define the *far* relation R by

$$xRy \iff d(x, y) > 1.$$

This relation gives a uni-modal logic of (X, d) which is known as the *farness logic* of (X, d), denoted by $\mathrm{Log}_{>1}(X)$. In this talk, I will present results on the farness logics of $\mathbb{R}^n$ and $\mathbb{Q}^n$, recently obtained in a joint work with Gabriel Agnew, Uzias Gutierrez-Hougardy, John Harding, and Ilya Shapirovsky [1].

Namely, we show that the farness logics of $\mathbb{R}^n$ are all distinct and contain an infinite anti-chain. In particular, for $n < m$, we have $\mathrm{Log}_{>1}(\mathbb{R}^m) \not\subseteq \mathrm{Log}_{>1}(\mathbb{R}^n)$. Additionally, for m sufficiently larger than n we have $\mathrm{Log}_{>1}(\mathbb{R}^n) \not\subseteq \mathrm{Log}_{>1}(\mathbb{R}^m)$. We also show that the logics $\mathrm{Log}_{>1}(\mathbb{R}^n)$ and $\mathrm{Log}_{>1}(\mathbb{Q}^n)$ are not finitely axiomatizable and $\mathrm{Log}_{>1}(\mathbb{Q})$ is strictly contained in $\mathrm{Log}_{>1}(\mathbb{R})$. Furthermore, $\mathrm{Log}_{>1}(\mathbb{R})$ lacks the finite model property.

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[1] Gabriel Agnew, Uzias Gutierrez-Hougardy, John Harding, Ilya Shapirovsky, and Jackson West, *On distance logics of Euclidean spaces*, arXiv:2501.04884, 2025.

▸ HONGYU ZHU, *The Borel complexity of the class of models of first-order theories.*
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Viewed as a subset of Cantor space, the class of countable models $\mathrm{Mod}(T)$ of any first order theory T is always Borel. A natural question, then, is the relationship between its descriptive complexity and the complexity of the underlying theory. Using theorems of

López-Escobar and Solovay, we give a more precise characterization of the complexity of $\mathrm{Mod}(T)$ in terms of that of T. We also discuss some applications to models of PA and infinitary logic. (This is based on joint work with Andrews, Gonzalez, Lempp, Rossegger [1], and related to recent work of Enayat and Visser [2].)

[1] Uri Andrews, David Gonzalez, Steffen Lempp, Dino Rossegger, and Hongyu Zhu, *The Borel complexity of the class of models of first-order theories*, arXiv:2402.10029, 2024.

[2] Ali Enayat and Albert Visser, *Incompleteness of boundedly axiomatizable theories*, ***Proceedings of the American Mathematical Society***, vol. 152 (2024), no. 11, pp. 4923–4932.

Abstracts of talks presented by title

▸ JOACHIM MUELLER-THEYS, *The monotonicity paradox and its solution.*
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I. It is a truism that laws must not be simply extended. *Substitution* is a metatheorem of propositional logic, which becomes false in our metalogical (or auto-modal repectively) extensions *MPL*: $\neg\Box p$ is logically true (since p can be interpreted false), but $\neg\Box\top$ is *not*. The same happens to *monotonicity*: $\neg\Box p$ is *no* consequence of p.

II. In contrast, we found and proved that *monotonicity belongs to the valid principles of universal semantics*: $\forall f \in S\ (F\ \mathrm{Seq}_T\, f\ \&\ G \supseteq F \Rightarrow G\ \mathrm{Seq}_T\, f)\ (F, G \subseteq S)$, whereby $\mathrm{Seq}_T \subseteq \wp(S) \times S$ ("consequence") is defined in standard, Tarskian manner: $F\ \mathit{Seq}_T\, f$ $:\Leftrightarrow \forall w \in W\,(F\,T\,w \Rightarrow f\,T\,w)$, but the underlying relation: $T \subseteq S \times W$ ("true at"), between $S \neq \emptyset$ ("*Sprache*") and $W \neq \emptyset$ ("worlds") is *free*.

It follows that *general consequences and logical laws coincide*: $\mathit{Seq}_\forall\, f :\Leftrightarrow \forall F\, F\, \mathrm{Seq}\, f$ $\Leftrightarrow \emptyset\ \mathrm{Seq}\, f \Leftrightarrow$: $\mathit{Seq}_0\, f \Leftrightarrow Vf :\Leftrightarrow \forall w\, f\, T\, w$.

III. How do I and II go together though? Let $\Im^*$ be the set of all interpretations $I(p) \in \{1, 0\}$. We extend PL satisfaction $\models\ \subseteq \Im^* \times L$ conservatively to $|\!\models_\Phi\ \subseteq \Im^* \times L^\Box$ $(\Phi \subseteq L)$, for $\alpha = \Box\beta$ specifically by $I \,||\!=_\Phi\, \alpha :\Leftrightarrow \forall J\,(J \models \Phi \Rightarrow J \,||\!=_\Phi\, \beta)$, and PL consequence $\Phi \models \phi$ to $\Phi \,||\!= \alpha :\Leftrightarrow \forall I\,(I \models \Phi \Rightarrow I \,||\!=_\Phi\, \alpha)$.

In analogy to II, $I \,||\!=_\Phi\, \alpha$ induces the consequence $\Gamma \,||\!=_\Phi\, \alpha$. The theorem of II now yields indeed that $||\!=_\Phi$ is monotonic, but it does *not* yield that $\Phi \,||\!=\, \alpha$ and $\Psi \supseteq \Phi$ generally imply $\Psi \,|\!\models \alpha$, since $\Phi \,|\!\models \alpha$ corresponds to $\Phi \,|\!\models_\Phi \alpha$, whereas $\Psi \,|\!\models \alpha$ corresponds to $\Psi \,||\!=_\Psi\, \alpha$. As indicated in I, concrete refutation is e. g. by $\Phi := \emptyset$, $\alpha := \neg\Box p$, $\Psi := \{p\}$.

Notes. This abstract is widely self-contained. There have been several précis in "The Bulletin of Symbolic Logic" as well on MPL (with some severe misprints) as on abstract semantics. In a letter to Wilfried Buchholz from March 2024, we, among other things, explained that monotonicity, despite of its universal validity, fails for auto-modal extensions, because satisfaction differs. We are grateful to anybody who has helped us.