

2022 WINTER MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Palmer House, Chicago, IL
Central APA Meeting
February 24, 2022

Program committee: Marcus Rossberg and Gila Sher (Chair).

The 2022 Winter Meeting will be held as part of the meeting of the Central Division of the American Philosophical Association. All ASL meeting participants must register for the APA conference. Registration is at www.apaonline.org/page/2022C_registrationinfo.

The Central APA Meeting runs February 23-26, 2022 and includes other talks and sessions of interest to logicians. The complete program will be available through the website of the Central Division of the APA.

Thursday February 24, 9:00 AM– 12:00 PM

ASL Plenary Session I

Chair: Timothy Bays

- 9:00 – 9:50 **Eileen Nutting** (Kansas) *Approaches to ordinal abstraction*
10:00 – 10:50 **Roy Cook** (Minnesota) *Notes towards a Kripke model of smooth infinitesimal analysis*
11:00 – 11:50 **Sean Ebels-Duggan** (Northwestern) *Vagueness, specificity, and mathematical structure*

Thursday February 24, 7:15 PM– 10:15 PM

ASL Plenary Session II

Chair: Eileen Nutting

- 7:15 – 8:05 **Sun-Joo Shin** (Yale) *Peirce’s triadic logic: extension or deviation?*
8:15 – 9:05 **Michael Titelbaum** (Wisconsin) *The logical firmament*
9:15 – 10:05 **Timothy Bays** (Notre Dame) *Tennenbaum’s theorem, induction, and the implicit definition of the natural numbers*

Abstracts of invited plenary lectures

- ▶ **ROY COOK**, *Notes towards a Kripke model of smooth infinitesimal analysis*.
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Smooth infinitesimal analysis (SIA) is an axiomatization of real analysis which includes axioms that guarantee the existence of nilsquares: infinitesimals so “small” that, although they fail to be identical to zero, their squares are identical to zero. These axioms are inconsistent if one works within classical logic, but SIA has been shown to be consistent within an intuitionistic setting via category-theoretic constructions. Unfortunately, the category-theoretic methods do not provide a good intuitive picture of what the SIA continuum “looks like”. Thus, in this talk I will construct Kripke models

for SIA (as well as a number of subtheories of full SIA) - models which make apparent the dynamic character of the SIA domain. The models in question, viewed from the (classical) metatheory, display both indeterminacy of identity and non-constancy of domain. Further, I will argue that the “intended” model of SIA (again, as seen from the classical metatheory), is, in a certain sense, countably infinite.

- ▶ SEAN EBELS DUGGAN, *Vagueness, specificity, and mathematical structure.*

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Color predicates, to take a well-worn example, are vague. This patch of blue is more purple than the second patch, but it is still blue. Keep this up and you’ll call purple things blue, which they are not. But of course we could add another word, and say that now we have blue, indigo, and purple. Adding “indigo” is an example of moving to a language with greater specificity. But of course, the trouble repeats. What is curious is that when characterizing vagueness, we often resort to further specification in order to emphasize the vagueness. “Imagine a color which is just a shade darker than our original sample”, etc. Additionally, in the case of color words, we are lucky, in that we can move to a language of maximal specificity: the language of real numbers, interpreted as reflective wavelengths. Something is lost in this move, of course, and it depends on the very happy accident that colors are determined by reflective wavelengths. This presentation will address several of the questions that arise in light of these observations. Is there always a language of greater (or even maximal) specificity available? Does a language of maximal specificity “disconnect” from the original language, in the way that the language of real numbers disconnects from the color language? In particular, we will address whether the structure of a dense linear order can be recovered from predicates exhibiting the kind of vagueness we observe in color terms.

- ▶ EILEEN NUTTING, *Approaches to ordinal abstraction.*

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There are two general approaches to ordinal abstraction. On the first, ordinals are abstracted from well-orderings. On the second, they are abstracted from elements positioned within such orderings. On either approach, avoiding the Burali-Forti Paradox requires setting restrictions on the application of ordinal abstraction. Several such restrictions have been proposed. I will offer an alternative that can be applied to ordinal abstraction on either approach, and I will argue that it is preferable to the restrictions in the literature. This alternative relies on Øystein Linnebo’s account of dynamic abstraction.

- ▶ SUN-JOO SHIN, *Peirce’s triadic logic: extension or deviation?*

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The talk has two goals: (i) to suggest a new way to understand Peirce’s triadic logic and (ii) to raise the question of whether Peirce’s triadic logic is an extension of or a deviation from classical logic. I classify Peirce’s six binary connectives, based on the dominance among three values, **V**, **F**, and **L**. Then, they may be grouped into three, depending on how the third value **L** is placed in the dominance hierarchy. (My visual representation makes the issue clearer.) Where is Peirce’s triadic logic located in a bigger picture, an extended standard logic or a non-standard logic? Traditionally, many-valued logic is non-standard, hence, a deviation from classical logic. While examining passages which seem to support the deviation view of Peirce’s new logic, I beg you to resist the temptation to rush to that conclusion. Peirce’s discussion on

the third value \mathbf{L} is rather nuanced. He says dyadic logic is defective (which is different from being incorrect) because dyadic logic “takes no heed of the *limit* between two realms.” Obviously, his new logic aims to cover the limit which traditional logic neglects to represent. Peirce draws our attention to the existence of “an intermediate ground” between both ends, i.e. “*positive assertion* and *positive negation*,” and wants to represent that middle area. According to this interpretation, Peirce’s triadic logic is an extension of classical logic, unlike other many-valued logics.

- MICHAEL TITELBAUM, *The logical firmament*.
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Rather than present new logical results, this talk concerns the metaphysics and epistemology of logic. How can we explain logical truths? When someone is ignorant of a logical truth, what specifically might they be ignorant of? Answers to these two questions usually focus on basic logical facts, like the definitions of connectives or simple valid inference patterns. But I identify another kind of fact, which I call a “catenary truth”, that may also figure in the answers. Previous epistemologies of logic ignore catenary truths, often to their detriment. I offer a new epistemology of logic that answers Benacerraf-style questions, avoids invoking intuitive faculties, and highlights an important contrast with other forms of a priori knowledge (such as moral knowledge).

Abstract of talk presented by title

- JOACHIM MUELLER-THEYS, *Concrete Existence*.
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The concrete way to demonstrate existence is the specification of some object having the property claimed. We can implement this natural conception into predicate logic by adding the *concrete existential quantifier* $\hat{\exists}$ being determined as follows: $(\mathcal{M}, V) \models \hat{\exists}x \phi$ iff there is a closed term t such that $(\mathcal{M}, V^x/t\mathcal{M}) \models \phi$, whereby $t^{\mathcal{M}} \in M$ is the usual interpretation of t by the structure \mathcal{M} (with domain M), $V(x) \in M$, and $V^x/a(y) := a$ if $y = x$; $V^x/a(y) := V(y)$ else. Dually, $\hat{\forall}x \phi := \neg \hat{\exists}x \neg \phi$. $\models \hat{\exists}x \phi \rightarrow \exists x \phi$, $\forall x \phi \rightarrow \hat{\forall}x \phi$.

Both converses are valid at *Named Logic*. We call \mathcal{M} *named* iff for all $a \in M$ there is a closed term t with $a = t^{\mathcal{M}}$. Any \mathcal{M} may be expanded to a named structure $\hat{\mathcal{M}}$.

Involving namedness, we could model-theoretically characterize the *morphisms* in a striking manner. We say that \mathcal{M} is *atomically entailed* by \mathcal{N} iff $\mathcal{M} \models \alpha$ implies $\mathcal{N} \models \alpha$ for all atomic sentences. Atomic entailment characterizes named structures already up to homomorphy, while atomic equivalence named characterizes named structures even up to isomorphy. Dealing with algebras, it suffices to regard closed equations.

All structures at all are *named axiomatized* by the basic theories of their named expansions, consisting from the atomic and negated-atomic sentences true. Formally, $\text{Th}_{\text{bas}}(\hat{\mathcal{M}}) \models \sigma$ iff $\mathcal{M} \models \sigma$. If the initial structure \mathcal{M} is named, it is evident and hence legitimate to regard but named models. So $\text{Th}_{\text{bas}}(\mathcal{M})$ completely axiomatizes \mathcal{M} then.

The major number systems starting with $(\mathbb{N}, 0, ')$ are named. In civilised countries, all citizens have official names. No set is named.

Given the concrete existential quantifier, named logic can be described by means of the axiom $\forall x \hat{\exists}y x \doteq y$, since $\mathcal{M} \models \mathbf{Named}$ iff \mathcal{M} is named. *Named consequence* $\Sigma \models \sigma$ is then alternatively given by $\hat{\Sigma} \models \sigma$, where $\hat{\Sigma} := \{\mathbf{Named}\} \cup \Sigma$.

Given concrete existence, *abstract existence* $\check{\exists}x \phi$ may be defined by $\exists x \phi \wedge \neg \hat{\exists}x \phi$.

Then, though $\mathcal{M} \models \phi[a]$ exists, there is no t with $\mathcal{M} \models \phi[t^{\mathcal{M}}]$. An example on the surface is $\exists x x \doteq x$ at languages with no descriptive symbols.

Concrete existence is a kind of constructive existence — a relationship that ought to be explored closer.

References. “The idea of Named Logic”, 2020 ASL Annual Meeting, Long Program: pp. 32-3, BSL 26 (2020): pp. 197-8; “Named Model Theory”, 2021 ASL-JMM, talk available, ASL-abstract (Full Program: pp. 5-6), AMS-abstract. The author believes that *Named Logic* has been joint work with Wilfried Buchholz ever since 1995. We are grateful to anyone who has helped us.