Logic in Africa: 17–19 May 2021

The Association for Symbolic Logic (ASL) Committee on Logic in Africa will host the first "Logic in Africa" event. It is a series of three online seminars that will be held each day at 5pm Central Africa Time (UTC +2).

<u>Monday 17 May:</u>

"Some classes of $\mathcal{RML}$ with collapsing multi-structures"
Celestin Lele (Université de Dschang, Cameroon)

<u>Tuesday 18 May:</u>

"Atom-canonicity in varieties of cylindric algebras with applications to omitting types in multi-modal logic"
Tarek Sayed Ahmed (Cairo University, Egypt)

<u>Wednesday 19 May:</u>

"Some applications of logic to the combinatorics of countable structures"
Rehana Patel (African Institute for Mathematical Sciences, Senegal)

To join the seminars, please use the following link (the same link will be used for all three days' talks):

Join Zoom Meeting `https://wits-za.zoom.us/j/93569764565`
Meeting ID: 935 6976 4565

The committee is grateful to the DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS) in South Africa for providing the Zoom hosting facilities.

ASL Committee on Logic in Africa:
Willem Conradie and Daoud Siniora (co-chairs), Andrew Craig, Moataz El-Zekey, Willem Fouché, Tarek Sayed Ahmed

Some classes of $\mathcal{RML}$ with collapsing multi-structures

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A residuated multilattice is a partially ordered commutative residuated monoid (a.k.a pocrim) whose poset is a multilattice. In other words, residuated multilattices (or $\mathcal{RML}$s for short) combine in a delicate manner the pocrim and multilattice structures on the same set. Therefore $\mathcal{RML}$s generalize both residuated lattices and multilattices. Cabrera et al. [1] laid the ground work on the topic by introducing the main properties and also studying filters within the new framework.

As a generalization of residuated lattices, it seems natural to consider various measures of the gap between the two types of systems. The main underlying question is the following: Which additional properties of $\mathcal{RML}$s would force the structure down to residuated lattices? For instance Cabrera et al. prove that an $\mathcal{RML}$ with idempotent product (i.e. $x \odot x = x$) is a Heyting algebra. We build on this and explore several equations some of which force the $\mathcal{RML}$ back to a subclass of residuated lattices, while others yielding new classes of $\mathcal{RML}$s that properly contain well-known classes of residuated lattices.

We consider several classes of residuated multilattices with an additional equation. We show that in some cases (e.g. for divisibility) the equation causes the multilattice structure to collapse while in other cases (semi-divisibility and regularity) we obtain examples attesting that these classes properly contain those of semi-divisible and regular residuated lattices respectively. We hope this work extends and deepens the foundations laid by Cabrera et al. and sets the stage for future studies on residuated multilattices.

References:

[1] I.P. Cabrera, P. Cordero, G. Gutiérrez, J. Martinez, M. Ojeda-Aciego, On residuation in multilattices: Filters, congruences, and homomorphisms, Fuzzy Sets and Systems **234** (2014) 1–21.

[2] M. Cornejo, J. Medina, E. Ramírez. A comparative study of adjoint triples. Fuzzy Sets and Systems **211** (2012) 1–14.

[3] U. Höhle, E.P. Klement, Non-classical logics and their applications to fuzzy subsets: A Handbook of the Mathematical Foundations of Fuzzy Set Theory, Springer Netherlands 32 (1995).

[4] J. Paralescu, Divisible residuated lattice of fractions and maximal divisible residuated lattice of quotients, Annals of the Alexandru Ioan Cuza University Mathematics, 1445, ISSN (Print) (2010) 1221–8421.

[5] E. Turunen, J. Mertanen, States on semi-divisible residuated lattices, Soft Computing **12** (2008) 353–357.

Atom-canonicity in varieties of cylindric algebras with applications to omitting types in multi-modal logic

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A variety V of Boolean algebras with operators is atom–canonical, if whenever $\mathfrak{A} \in \mathsf{V}$ is atomic, then its Dedekind–MacNeille completion, sometimes referred to as its minimal Monk completion, is also in V. Fix $2 < n < \omega$. Let L_n denote first order logic with n variables, and let RCA_n, its semantic algebraic counterpart, denote the class of representable cylindric algebras of dimension n. We show that several varieties (in fact infinitely many) containing and including the variety RCA_n are not atom-canonical. This adds to the complexity of potential axiomatizations of RCA_n; any necessarily infinite such equational axiomatization cannot consist exclusively of Sahlqvist equations. From our hitherto obtained algebraic results we show, using games, graph theory and combinatorics, that the celebrated Henkin–Orey omitting types theorem, briefly OTT, which holds for $L_{\omega,\omega}$, fails dramatically for L_n even if we allow certain generalized models that are only locally classical on '$m \leqslant \omega$ squares'—with $n \leqslant m \leqslant \omega$ and ω-squareness coinciding with ordinary models. Given $2 < n < l < m \leqslant \omega$ an m-square model is l-square, but the converse is not true. Let $2 < n \leqslant l < m \leqslant \omega$ and let $\psi_n(l,m)$ be the following statement stipulating failure of OTT: There exists a complete countable atomic L_n theory T (meaning that the Lindenbaum–Tarski quotient algebra $\mathfrak{Fm}_T$ is atomic) and a type Γ such that Γ is realizable in every m-square model of T, *a fortiori* any model of T, but Γ cannot be isolated using l variables. We prove $\psi_n(l,\omega)$ for every $2 < n \leqslant l < \omega$, and $\psi_n(n, n(n+1)/2+1)$. We deduce that OTT and the modal property of di-persistence fails for the multi-modal logic $\mathsf{S5}^n$ and the so-called clique guarded (equivalently packed) fragments of L_n. In contrast, we show that if T is a countable L_n theory that admits elimination of quantifiers, then $< 2^{\aleph_0}$ many non-principal complete types of T, can be omitted.

A key result in the work presented in my talk is joint with Hajnal Andréka and Istvan Németi [1].

References:

[1] T.S. Ahmed, H. Andréka, and I. Németi, Omitting types for finite variable fragments and complete representations of algebras. Journal of Symbolic Logic **73**, (2008) 65–89.

Some applications of logic to the combinatorics of countable structures

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Logical methods have had wide-ranging applications in the study of countable combinatorial structures. I will describe three in which I have had a hand: on tame regularity phenomena, big Ramsey degrees, and exchangeable constructions. The talk will be self-contained, and all definitions will be given. This is joint work with Ackerman and Freer, and with Coulson and Dobrinen.