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OF THE ASSOCIATION FOR SYMBOLIC LOGIC

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Abstracts of invited papers

▸ PATRICIA BLANCHETTE, *Axioms, models and theories: from geometry to logic.*
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As the methods and concepts of modern logic emerged in the early 20th century, the crucial concepts of *axiom*, of *model*, and even of *theory* went through a conceptual transformation, with the result that none of these concepts plays the same role in theories of pure logic by 1920 that it played in theories of geometry or arithmetic a few decades earlier. As a result, the claims we can now demonstrate about our theories (e.g., consistency of various kinds, entailment and independence of various kinds) do not answer the same questions one might have asked about theories just shortly before the turn-of-the-century revolution. This talk explores some of the important differences between old and new, with the goal of shedding some light on the kinds of things we can, and the kinds of things we cannot, demonstrate via modern methods.

▸ DAVID DEVIDI, *Intermediate logics and metaphysics: On what there is, what there isn't, and none of the above.*
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It is a philosophical commonplace that logic and metaphysics have been closely related disciplines from the beginning.The close relationship has survived, and indeed thrived, throughout the rapid evolution and diversification of logic over the past 150 years.

Famously, there were rocky stages in which logic was thought to be the key to rubbing out metaphysics, but the focus in this talk will be on the idea that the tools of formal logic allow us to illuminate, rather than eliminate, our metaphysical commitments. I will suggest that certain results in some non-classical logics (in particular, logics intermediate in strength between intuitionistic and classical logic, and results about the additions to intuitionistic logic that give rise to them) have not yet received due consideration in discussions of metaphysics. I will also suggest that some recent discussions by, for instance, Norbert Gratz and Georg Shiemer, about different ("Hilbertian" and "Russellian") sorts of indeterminacy, are more illuminating if viewed through non-classical rather than classical logical lenses. The goal is a more nuanced picture of our metaphysical options, from varieties of structuralism to more interesting answers to questions about what is real, what is not, and what the other options are.

▸ J. MICHAEL DUNN, *Two, three, four, infinity: The path to the Belnap–Dunn four-valued logic.*
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My talk is loosely based on my contribution under a similar title to the forthcoming book *New Essays on the Belnap–Dunn Logic*, Heinrich Wansing and Hitoshi Omori (eds.), Synthese Library, Springer. I will give a kind of intellectual history of the so-called Belnap–Dunn Four-valued Logic, examining its evolution: the 4-element De Morgan lattice of Antonio Monteiro, Timothy Smiley's 4-element matrix for Belnap's Tautological Entailment, Dunn's interpretation in terms of "aboutness," Bas van Fraassen's semantics for Tautological Entailment using "facts," and Dunn's interpretation in terms of how a sentence can be both true and false, or neither true nor false, as well as the usual two values simply true, or simply false. Of course, I will discuss Nuel Belnap's viewing the four values as elements in a "bi-lattice" (under the influence of Dana Scott) and Belnap's famous use of the four-valued interpretation for "How a computer should think." I shall also examine relationships to Richard Routley and Valerie Routley's "star semantics;" I will then examine later adaptations and extensions of this idea, including work by Yaroslav Shramko, Tatsutoshi Takenaka, Dunn, Wansing, Omori on "trilattices." I will also explain my recent extension to an infinite valued "Opinion Tetrahedron" which has the four values as its apexes. I will discuss the application of this to inconsistencies on the World Wide Web, and elsewhere.

▸ NORBERT GRATZL, *Classical logic as single conclusion & logicality.*
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Classical Logic is one of the 20th century success stories. Having said this there is no lack of criticism since classical logic has been scathed for many different reasons: it does not account for ordinary reasoning processes, it is not constructive, (in some formulations) it allows for multiple conclusions, (its material) implication is paradoxical, it explodes—i.e., from a contradiction every formula (of some language) follows, it is monotone, its consequence relation is irrelevant, it is not in accord with our best inferential practice. Of course not every objection to Classical Logic can be rebutted. This holds, e.g., for the paradoxes of material implication or the monotonicity of Classical Logic.

Proof-theoretic semantics (PTS) is by now a well established field of research. One of the goals of proof-theoretic semantics is to account for a theory of meaning of the logical operators. Among the constraints of PTS is that a logic should be single conclusion. Some authors argue that logic *should* remain single, e.g., [3].

First, two extensions of Intutionistic Logic have been presented in [2] and [5]. For the point of view of proof-theoretic semantics, these proposed solutions can be challenged.

Second, our proposal builds on an extension of minimal logic by use of two meta-rules of inference. This calculus is provably equivalent to classical logic (in Hilbert-style) and does not enjoy cut elimination. There is, however, a remedy for this. The remedy consists of an auxiliary calculus that contains just enough classical principles to obtain full classical logic and enjoys cut elimination. A side-effect of the auxiliary calculus is that it allows for a measure of non-constructivity. Roughly put the basic idea is how strong the end-sequent is from the standpoint of minimal logic. Furthermore this auxiliary calculus serves another purpose, i.e., investigating logicality. Thereby ideas from K. Dosen [1] are put to use which are suitably adapted for this logical investigation.

[1] K. Dosen, *Logical constants as punctuation marks*, ***Notre Dame Journal of Formal Logic***, vol. 30 (1989), pp. 362–381.

[2] S. Negri and J. von Plato, ***Structural proof theory***, Cambridge University Press, Cambridge, 2008.

[3] F. Steinberger, *Why conclusions should remain single*, ***Journal of Philosophical Logic***, vol. 40 (2011), pp. 333–355.

[4] N. Tennant, ***Core Logic***, Oxford University Press, Oxford, 2017.

[5] J. Von Plato and A. Siders, *Normal derivability in classical natural deduction*, ***The Review of Symbolic Logic***, vol. 5 (2012), no. 2, pp. 205–211.

[6] H. Wansing, ***Display modal logic***, Kluwer, Dordrecht, 1998.

▸ HANNES LEITGEB, *HYPE: A system of hyperintensional logic.*
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This lecture introduces, studies, and applies a new system of logic which is called 'HYPE'. In HYPE, formulas are evaluated at states that may exhibit truth value gaps (partiality) and truth value gluts (overdeterminedness). Simple and natural semantic rules for negation and the conditional operator are formulated based on an incompatibility relation and a partial fusion operation on states. The semantics is worked out in formal and philosophical detail, and sound and complete axiomatizations are provided both for the propositional and the predicate logic of the system. The propositional logic of HYPE can be shown to contain first-degree entailment, to have the Finite Model Property, to be decidable, to have the Disjunction Property, and to extend intuitionistic propositional logic conservatively when intuitionistic negation is defined appropriately by HYPE's logical connectives. Furthermore, HYPE's first-order logic is a conservative extension of intuitionistic logic with the Constant Domain Axiom, when intuitionistic negation is again defined appropriately. The system allows for simple model constructions, and its logical structure matches structures well-known from ordinary mathematics, such as from optimization theory, combinatorics, and graph theory. HYPE may also be used as a general logical framework in which different systems of logic can be studied, compared, and combined. In particular, HYPE is found to relate in interesting ways to classical logic and to various systems of relevance and paraconsistent logic, many-valued logic, and truthmaker semantics. On the philosophical side, if used as a logic for theories of type-free truth, HYPE is shown to address semantic paradoxes such as the Liar Paradox by extending non-classical fixed-point interpretations of truth by a conditional as well-behaved as that of intuitionistic logic. Finally, just as classical models may be extended to a possible worlds semantics for modal operators that create intensional contexts, HYPE-models may be extended to a possible states semantics for modal operators that create hyperintensional contexts. In this way, the logic of HYPE may serve as a background logic that applies in the scope of certain hyperintensional operators (e.g., causal operators of the form 'A causes B').

[1] H. Leitgeb, *HYPE: A System of Hyperintensional Logic*, ***Journal of Philosophical Logic***, forthcoming.

▸ PAOLO MANCOSU, *Neologicist foundations: inconsistent abstraction principles and part-whole.*
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Neologicism emerges in the contemporary debate in philosophy of mathematics with Wright's book Frege's *Conception of Numbers as Objects* (1983). Wright's project was to show the viability of a philosophy of mathematics that could preserve the key tenets of Frege's approach, namely the idea that arithmetical knowledge is analytic. The key result was the detailed reconstruction of how to derive, within second order logic, the basic axioms of second order arithmetic from Hume's Principle

$$\text{(HP)} \quad \forall C, D\left(\#(C) = \#(D) \leftrightarrow C \cong D\right)$$

(and definitions). This has led to a detailed scrutiny of so-called abstraction principles, of which Basic Law V

$$\text{(BLV)} \quad \forall C, D\left(\mathit{ext}(C) = \mathit{ext}(D) \leftrightarrow \forall x\,(C(x) \leftrightarrow D(x))\right)$$

and HP are the two most famous instances. As is well known, Russell proved that BLV is inconsistent. BLV has been the only example of an abstraction principle from (monadic) concepts to objects giving rise to inconsistency, thereby making it appear as a sort of monster in an otherwise regular universe of abstraction principles free from this pathology. We show that BLV is part of a family of inconsistent abstractions. The main result is a theorem to the effect that second-order logic formally refutes the existence of any function F that sends concepts into objects and satisfies a part-whole relation. In addition, we study other properties of abstraction principles that lead to formal refutability in second-order logic.

This is joint work with Benjamin Siskind (UC Berkeley).

▸ EDWIN MARES, *Theories of entailment.*
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There have been various approaches to understanding entailment as the "converse of deducibility." Perhaps, best known are the "reflexive approach" due to Dana Scott and Kosta Došen and the semantic approach appealing to possible worlds. This paper takes a different tack, treating a logic of entailment as determining a Tarski-style consequence operator. Generalizing this idea and looking at some intuitive properties of such operators motivates certain relevance logics, such as DJ and Anderson and Belnap's logic E of relevant entailment.

▸ GREG RESTALL, *Generality and existence 2: Modality and quantifiers.*
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In this talk, I motivate and define a cut free sequent calculus for first order modal predicate logics, allowing for singular terms free of existential import. I show that the cut rule is admissible in the cut-free calculus, and I explore the relationship between contingent 'world-bound' quantifiers and possibilist 'world-unbound' quantifiers in the system.

The paper on which this talk is based is a sequel to "Generality and Existence 1: Quantification and Free Logic" [1].

[1] Greg Restall, *Generality and existence 1: Quantification and free logic*, ***The Review of Symbolic Logic***, to appear.

▸ ALASDAIR URQUHART, *Desargues's theorem in relevance logic.*
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This talk surveys the uses of Desargues's theorem in relevance logics. The applications include undecidability results as well as the proof of the failure of interpolation.

Abstract of contributed papers

▸ DOUGLAS BLUE, *Mathematically proper evidence revisited.*
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Martin characterizes *mathematically proper evidence* as *a priori* "evidence that counts towards giving mathematical knowledge of the truth of a proposition" [1, p. 219]. The character of his examples is essentially prediction and confirmation. For example, using the Axiom of Determinacy, Martin proved the Cone Lemma. Later, the Cone Lemma for Borel sets was confirmed in ZFC with his proof of Borel Determinacy.

We identify a different sort of mathematically proper evidence consisting of pairs of theorems (ϕ, ϕ^*) such that (1) ϕ is provable from ZFC+Large Cardinals and is best possible in this theory, and (2) ϕ^* is provable in ZFC+Large Cardinals+V = Ultimate-L and is the best possible strengthening of ϕ. We argue that this class of evidence suggests a conception of mathematically proper evidence whereby theory turns theorems into evidence. To illustrate this conception, we focus on one pair of theorems, the HOD Dichotomy Theorem of Woodin [3] and the well-known consequence of the axiom V = Ultimate-L that V = HOD, and the philosophical significance it has for the view that the definable sets are "far from V," a view encapsulated by the following passage in [2]: "Some would go further and claim disbelief that the real line can be definably wellordered on any rank—it is quite plausible that the only sets of reals which admit definable wellorderings are countable" [p. 472].

[1] Donald A. Martin, *Mathematical evidence*, ***Truth in mathematics*** (Harold G. Dales and Gianluigi Oliveri, editors), Oxford University Press, Oxford, England, 1998, pp. 215–231.

[2] Yiannis Moschovakis, ***Descriptive set theory***, American Mathematical Society, 2009.

[3] W. Hugh Woodin, *In search of Ultimate-L*, ***The Bulletin of Symbolic Logic***, vol. 23 (2017), no. 1, pp. 1–109.

▸ RACHEL BODDY, *Fregean definitions in the philosophy of mathematics.*
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How can definitions play an explanatory role? And how can definitions be used to address philosophical questions? Frege famously tried to solve philosophical questions about mathematics by providing logical proofs. In this paper, I argue that Frege's strategy depended on a particular view of definitions, and the sense in which definitions can afford explanations. I then use this discussion to critically examine the neo-Fregean proposal to use Hume's Principle as an implicit definition of number, and to found arithmetic on its basis. I argue that this proposal presupposes an answer to the above two questions that is distinctively *un*-Fregean, and that it conflates two notions of definition, viz., one concerned with conceptual analysis, the other concerned with the construction of gap-free proof, which Frege was careful to keep apart. This highlights an important, yet often overlooked, way in which Frege's perspective differs from that of contemporary neo-Fregeans.

▸ NICHOLAS FERENZ, *Identity in the relevant logic R.*
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Philip Kremer [2] argues for an axiomatization of the logic **R** with identity, which formalizes what he calls the *relevant indiscernibility interpretation of identity*. This axiomatization is motivated by its *stability*/internal coherence and its relation to Dunn's relevant predication [1]. Building on the semantics for quantified relevant logic of Mares and Goldblatt [3], I construct semantics for Kremer's axiomatization and a few related systems. Additionally, extending the semantics to other relevant logics with identity is considered.

[1] J. Michael Dunn, *Relevant predication 1: The formal theory*, ***Journal of Philosophical Logic***, vol. 16 (1987), no. 4, pp. 347–381.

[2] Philip Kremer, *Relevant identity*, ***Journal of Philosophical Logic***, vol. 28 (1999), no. 2, pp. 199–222.

[3] Edwin D. Mares and Robert Goldblatt, *An alternative semantics for quantified relevant logic*, ***The Journal of Symbolic Logic***, vol. 71 (2006), no. 1, pp. 163–187.

▸ RONALD FULLER, *Non-verbal communication in classical logic.*

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Uninterpreted logical structures are generally considered to have no meaning. But in disciplines of applied logic, such as information management and software engineering, uninterpreted structures are meaningful. Put another way, logical vocabularies and sentences can express meaning even when the value of object constants are unknown, incorrect or absent. Meaning can be conveyed by the mode of presentation alone. Frege called this *sense*, Carnap called it *intension*.

The Stanford Encyclopedia of Philosophy says outright "In classical first-order logic intension plays no role." This is misleading. It is true that no awareness of meaning is needed for a person or machine to analyze uninterpreted statements, or to create derived statements from them. And it is true that many such statements are indeed meaningless. Logicians use meaningless structures to teach and study logic. But when formed properly and meaning is intended they do convey meaning. When formed *improperly* and meaning is intended they convey an approximation or guess at the intended meaning, which can lead to problems.

Modern institutions rely heavily on disciplines of applied logic. For example, many institutions organize information into relational databases, which are based on logic. Leaders often want to combine or compare information between organizations but cannot because the metalanguage descriptions of similar relations are different, so some intended statements cannot be derived.

Logic literacy is necessary to convey any intended meaning using logic. The decline of logic education during the 20th century and the ensuing decline of logic literacy is the root cause of many difficult and costly challenges facing modern organizations.

▸ WESLEY FUSSNER AND SARA UGOLINI, *Monoidal t-norm based logic: a duality-theoretic perspective.*

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In lattice theory, triples constructions date back to Chen and Grätzer's 1969 decomposition theorem for Stone algebras: each Stone algebra is characterized by the triple consisting of its lattice of complemented elements, its lattice of dense elements, and a homomorphism associating these structures. A dual analogue of this construction was provided by Priestley in 1974 [3]. Later on, triples decompositions have been extended to richer algebraic structures. For example, [2, 1] provided similar triples decompositions for classes of MTL-algebras, the algebraic semantics of monoidal t-norm based logic. Our aim is to provide a duality theoretic perspective on these recent innovations. Our dualized construction gives a uniform way of building the extended Priestley spaces of a large class of MTL-algebras from the Stone spaces of their Boolean skeletons, the extended Priestley spaces of their radicals, and a family of nuclei connecting the two. Moreover, we present novel results regarding the extended Priestley duals of MTL-algebras, emphasizing their structure as Priestley spaces enriched by a partial ternary operation. This also yields a novel duality for generalized MTL-algebras, where the aforementioned partial operation is total.

[1] S. Aguzzoli, T. Flaminio, and S. Ugolini, *Equivalences between subcategories of MTL-algebras via Boolean algebras and prelinear semihoops*, ***Journal of Logic and Computation***, vol. 27 (2017), no. 8, pp. 2525–2549.

[2] F. Montagna and S. Ugolini, *A categorical equivalence for product algebras*, ***Studia Logica***, vol. 103, (2015), no. 2, pp. 345–373.

[3] H. A. Priestley, *Stone lattices, a topological approach*, ***Fundamenta Mathematicae***, vol. 84 (1974), pp. 127–143.

▸ NIKOLAOS GALATOS, *Skew reflections.*
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The upwards skew reflection of a Dunn monoid has been considered in relevance logic. We present a downward skew reflection that embeds a residuated lattice into an involutive one and revisit the upward skew reflection in the context of residuated lattices.

▸ FABIO LAMPERT, *A completeness theorem for n-dimensional modal logic.*
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There are many logical systems known as multidimensional modal logics. In particular, a variety of multidimensional modal logics has been used extensively to provide a logical analysis of necessity and a priori knowledge. For this purpose, two-dimensional modal logics, with two necessity operators, namely, one for metaphysical necessity, and one (diagonal) for the a priori, has occupied the center of investigation. These logics are known to be complete and finitely axiomatizable. But what about their generalizations from two to n-dimensions: Are such modal logics complete? Are they finitely axiomatizable? In this presentation we provide a positive answer for both questions.

▸ JOACHIM MUELLER-THEYS, *Specifying computability more naturally ?*
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Real computers work with logic gates (like NOR) implementing truth functions.

Computability has been defined by means of μ-recursive functions, Turing and register machines, λ-calculus, and in other ways.

We wonder whether and how a more natural characterization can be given that focuses on truth functions.

▸ JAMES WALSH, *Artificial languages and the determinacy of mathematical concepts.*
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Prima facie, arithmetical claims like "there are infinitely many primes" have an intended interpretation; they concern the structure $\mathcal{N} = \langle \mathbb{N}, 0, 1, +, \times \rangle$. However, a well-known skeptical argument inspired by Skolem allegedly shows that there is no intended interpretation of arithmetical discourse. This argument appeals crucially to a result in mathematical logic, namely, the existence of non-standard models of arithmetic. How can results in mathematical logic be brought to bear on mathematical practice, broadly construed to include the semantics of mathematical language?

I argue that mathematical logic is informative with respect to mathematical practice insofar as mathematical logic involves the study of scientific models of mathematical practice. Though the idealizations in these models facilitate the study of a narrow range of features of mathematical practice, they distort other features. I argue that these distortions motivate the aforementioned skeptical arguments. That is, the skeptical arguments turn on a conflation between mathematical practice and the scientific models of mathematical practice studied in mathematical logic.

▸ LAWRENCE S. WANG, *Wittgenstein and Carnap on Gödel's ontology.*
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Wittgenstein's criticism of Gödel, especially regarding the incompleteness theorems, are widely held as founded on a fundamental mathematical misunderstanding. Wittgenstein's interpretation

is largely inaccurate as an impredicative reduction, but remain substantial diagnoses of Gödel's stringent ontology. The central trouble is the notorious paragraph in *Remarks on the Foundations of Mathematics* that a proposition P in Russell's system interpreted P is not provable in Russell's system disintegrates: either P is proven the case ($\Rightarrow\Leftarrow$), or a provable $\neg$P obtains from the natural-language and logical consequence of P-unprovability.

The broader critique leveled in Remarks bears similarities to Carnap's disagreement with Gödel. Gödel's hyperbolic reading of Carnap identifies a broad language, including, i.e., mathematical interpretation as inherently syntactical, thus hypothetically constructible through syntax alone. Though Gödel disputes this hypothetical capacity, his ambivalent criticism reveals a strong tendency for a native inherent necessity to reconcile his own mathematical ontology, specifically about induction. Precisely this necessity is problematic for Wittgenstein, whose *Remarks* cash out the consequence of incompleteness as the ontologically strong, necessary syntactical element that Gödel's construction of an axiomatic system requires.

Wittgenstein demonstrates that the necessity derived from seemingly inherently syntactical constructions in the vein of P demands a content in the Gödel interpretation which is basic. The Remarks demonstrate the LEM between (P,$\neg$P) not as platonistic, but a manner of instrumentalizing the strict mathematical necessity in Gödel's thinking to illustrate a disjoint with an actual modal necessity; Gödel cannot maintain both serious consideration of consequences of mathematical facts, and the necessity of mathematical syntax. Either one must give up the language of consequence for a weaker ontology, or else explicit arbitrate a linkage between the syntactical necessity he has ascribed to mathematics and a question-begging platonic necessity of mathematical entities.

Abstracts of papers presented by title

▸ JOHN CORCORAN AND JAMES CARGILE, *Teaching counterexamples and counterinstances.*
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As in Tarski's [3], "variable-enhanced" English [1] translates into a first-order language interpreted in the [real] numbers.

The universal sentence [A] "for every number x, x is rational" has the number p as a counterexample [2]. Every number that isn't rational is a counterexample for A. Conversely, every counterexample for A is a number that isn't rational. Counterexemplification, expressed 'is a counterexample for', is a semantic relation of numbers to universal sentences.

The sentence [Z] "zero is rational" is an *instance* of A [3, p. 33], as is every sentence obtained from Z by replacing the name "zero" with any number name. An *instance* of A is a sentence obtained by deleting "for every number x" and replacing the remaining "x" with a number name—a purely syntactic operation [3, p. 47]. *Instantiation*, expressed by 'is an instance of', is a syntactic relation of sentences to sentences defined by syntactic means.

Every universal sentence implies each of its instances. But not every universal sentence is implied by the set of its instances—a point not found in [3].

The sentence [P] "π is rational" is a *counterinstance* of the universal sentence A, as is every sentence obtained from P by replacing the name π with any other name of a non-rational number. A *counterinstance* of a universal sentence is a false instance. *Counterinstantiation*, expressed by 'is a counterinstance of', is also a syntactic relation of sentences to sentences—but this definition used the semantic concept falsehood.

Every universal sentence contradicts each of its *counterinstances*. A corollary is that the negation of any universal sentence is implied by each of its counterinstances.

In this interpreted language, the counterexemplification relation relates numbers to universal sentences. The counterinstantiation relation relates sentences to sentences. Every counterinstance for a universal sentence contains a name of one of that sentence's counterexamples. Neither relation is discussed in [3].

[1] JOHN CORCORAN, *Variable-enhanced English and structural ambiguity*, ***The Bulletin of Symbolic Logic***, vol. 24 (2018), forthcoming.

[2] JOHN CORCORAN, *Counterexamples and proexamples*, ***The Bulletin of Symbolic Logic***, vol. 11 (2005), p. 460.

[3] ALFRED TARSKI, *Introduction to logic and to the methodology of deductive sciences*, Fourth edition, (Jan Tarski, editor), Oxford University Press, 1994.

▸ JOHN CORCORAN AND WILLIAM FRANK, *Tarski's quoted-expressions.*
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Quoted-expressions are character-strings beginning and ending with matched quotes (this BULLETIN, vol. 19 (2013), p. 131 and pp. 253–254). Deleting the first and last characters produces their *interiors*. Quoted-expressions frequently denote their interiors but they have many other uses. See our *Tarski's quoted-letters*, this BULLETIN, vol. 19 (2013), pp. 247–248, treating quoted-expressions whose interiors are single letters.

We survey Tarski's quoted-expression uses.

Tarski uses quoted-expressions whose interiors are quoted-expressions as in [1, pp. 159–160]:

The three-character string " 'p' " denotes one letter.

The above sentence contains the five-character quoted-expression ' " 'p' " ' denoting a three-character string " 'p' " denoting a one-character string, 'p', a letter denoting nothing.

No string serves as an *autonym* denoting itself [2, p. 344]; [3, p. 104].

Tarski also used quoted-letters as sentence-names: quoted-letters containing letters abbreviating sentences denote—not letters—but sentences [2, p. 347]; [3, p. 108].

's' is the sentence printed in this paper [. . .].

"s" is a sentence.

In [1, pp. 152–278], "functions" contain "free variables". The *quotation-function* ' 'x' ' is instantiated by expression-names: '1 'a' ', ' 'b' ', etc. Tarski noted that the quoted-letter ' 'x' ' is "ambiguous" (sic): sometimes it *is* a *function*—not a name; sometimes it is a letter-name—not a function [1, p. 162].

Another usage is in the truth-schema [3].

"p" is true if and only if p

Here the quoted-letter is neither a function nor a name: it contains 'p' as a place-holder (schematic letter) co-ordinate with a place-holder not part of a quoted name (this BULLETIN, vol. 19 (2013), pp. 507–508).

Other uses supplement the six above. In [1, p. 61] Tarski defined the cardinal $\#A$ of a set A as follows.

'$\#A$' denotes the cardinal of the set A.

Compare this to:

For every set A, $\#A$ is the cardinal of the set A.

[1] ALFRED TARSKI, ***Logic, semantics, metamathematics***, Hackett, 1956/1983.

[2] ———, *Semantic conception of truth*, ***Philosophy and Phenomenological Research***, vol. 4 (1944), pp. 341–375.

[3] ———, *Truth and proof*, ***Scientific American***, June, 1969.

▸ JOHN CORCORAN AND KEVIN TRACY, *Jan Tarski 1994 on two excluded-middles and two non-contradictions.*

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The late Alfred Tarski's classic masterpiece [3] originated in Polish in 1936 and appeared in German in 1937 and in English in 1941. Its merits were immediately recognized.

Alonzo Church [1] wrote that the chapter *On the deductive method* "treats the methodology of mathematics, postulate theory, in a way which altogether supersedes more familiar, but less accurate, accounts". Church concluded: "[...] a work of unusual merit, serves a purpose no other treatise adequately fulfills. It will have a great influence."

Haskell Curry's review [2] states: "This book and preceding editions were reviewed in several places. All reviewers, including the present one, are agreed that this is a work of exceptional merit [...] a masterpiece of exposition."

Versions appear in over eleven languages. The fourth edition edited by Alfred's son Jan contains corrections by Alfred, other revisions due to Jan and others, and supplementary notes by Jan. This presentation analyzes one of Jan's notes.

In 1941 Alfred presented two "excluded-middles", one for sentences, one for classes; likewise two "non-contradictions" [3, pp. 38 and 73].

$$\text{For any sentence } p,\ p \vee \sim p.$$

$$\text{For any sentence } p,\ \sim (p \wedge \sim p).$$

$$\text{For any class } K,\ K \cup K' = \vee.$$

$$\text{For any class } K,\ K \cap K' = \wedge.$$

In 1994 Jan noted that the last two laws resemble laws of sentential calculus and added that one "can easily verify that the above class-theoretical laws are direct consequences" of the two sentential-calculus laws [3, p. 73].

Our investigation leads beyond Jan's note to a study of the role Alfred attributed to sentential calculus in mathematical proof.

[1] Alonzo Church, Review: Alfred Tarski, ***Introduction to logic [...]***, ***The Journal of Symbolic Logic***, vol. 6 (1941), pp. 30–32.

[2] Haskell Curry, Review: Alfred Tarski, ***Introduction to logic [...]***, ***Bulletin of the American Mathematical Society***, vol. 48 (1942), pp. 507–510.

[3] Alfred Tarski, *Introduction to logic and to the methodology of deductive sciences*, Fourth edition, (Jan Tarski, editor), Oxford University Press, 1994.